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*ACCESS PRICING AND COMPETITION*

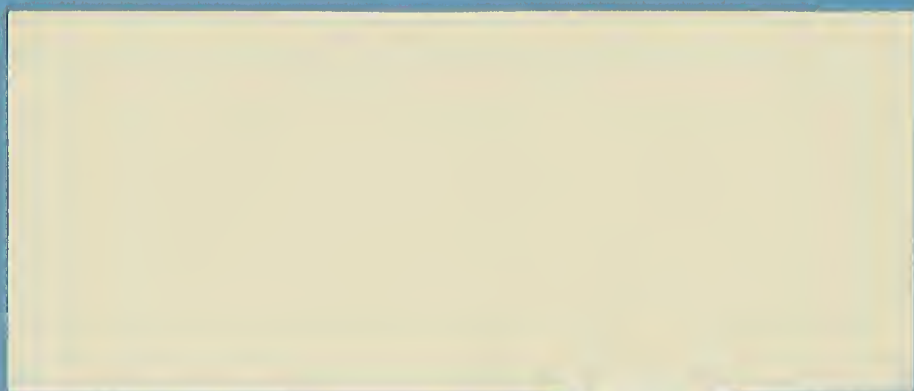
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94-31

July 1994

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***ACCESS PRICING AND COMPETITION***

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# Access Pricing and Competition\*

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### **Abstract**

In many industries (electricity, telecommunications, railways), the network can be described as a natural monopoly. A central issue is how to combine the necessary regulation of the network with the organization of competition in activities which use the network as an input and are potentially competitive (generation of electricity, value-added services, road transportation).

In this paper we derive access-pricing formulas in the framework of an optimal regulation under incomplete information. First, we study how access-pricing formulas take into account the fixed costs of the network and the incentive constraints of the natural monopoly over the network. Second, we examine the difficulties coming from the accounting impossibility to disentangle costs of the network and costs of the monopoly's competitive goods. Third, the analysis is extended to the cases where governmental transfers are prohibited, where the competitors have market power, and where competitors are not regulated. Fourth, the possibility some consumers bypass the network is taken into account. Finally the role of access pricing for inducing the best market structure is assessed. The conclusion summarizes our results and suggests avenues of further research.

# 1 Introduction

In many industries (electricity, telecommunications, railways), the network can be described as a natural monopoly. However, many activities which use the network as an input are potentially competitive (generation of electricity, value added services, road transportation). A central issue is how to combine the necessary regulation of the network with the organization of competition in those activities.

Two different types of policies can be observed. In the USA, the local network monopolies in the telephone industry have been prevented from entering the value added markets as well as the long distance market because of the Department of Justice's belief that it is impossible to define access rules to the network which create fair competition in those markets<sup>4</sup>. The argument here is that it is too easy for the monopoly in charge of the network to provide unfair advantages to its own products (favorable access charges, superior quality of access, R & D subsidies...) even if subsidiaries are created to improve cost auditing. In the language of modern regulatory economics, asymmetries of information between the regulator and the monopoly are so large that fair competition is not possible.

The alternative policy of defining access charges and letting the monopoly compete is also common. The long distance operator Mercury pays access charges to British Telecom (BT) to reach consumers through the local loops and compete with BT in the long distance market. The regulatory choice of the access charges is complex. For instance, in the regulatory review of 1991, BT has argued that the current setting of access charges creates unfair advantages for its competitor (Cave (1992)). Note also that it is the goal of the European Community to define "reasonable" access charges to organize the competition of electricity generation in Europe. Other examples of activities with regulated access are computer reservation systems and gas pipelines.

It is difficult to appraise the difficulties of regulating access if all the imperfections of regulatory processes are taken into account simultaneously. In this paper, we start from an optimal benevolent regulation of a network monopoly facing a competitive fringe in the potentially competitive markets and we successively introduce various imperfections. To focus on access we neglect the relevant issue of network externalities, which could be easily incorporated into our modeling. Perhaps more importantly, we ignore the divestiture policy. The modeling of the costs and benefits of vertical integration is complex, and, in a first step, we choose to focus on the vertically integrated structure as a building block of a more general theory in which breakups might be desirable.

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<sup>4</sup>"DOJ... did not believe that judicial and regulatory rules could be effective in preventing a monopolist from advantaging an affiliate that provided competitive products and services" Noll (1989).

The existing theoretical literature on network access pricing includes two papers building on contestable markets theory. Willig (1979) studies interconnection by competing suppliers. He stresses the fact that strategic behavior may lead the network operator to deter socially beneficial entry (for example to expand its rate base under rate of return regulation) and analyses the extent to which Ramsey pricing is compatible with the best market structure. Baumol (1983) discusses similar issues in the framework of railroad access pricing. Vickers (1989) addresses the access pricing issue within the Baron-Myerson model of regulation under incomplete information. He compares in particular a vertically integrated monopoly (network plus services) with the case of an imperfectly competitive sector of services with and without participation of the network operator.

Section 2 describes the model. A monopoly operates a network. It produces a monopolized commodity (for concreteness, local telephone), as well another commodity (long distance telephone) which faces a competitively produced imperfect substitute. The long distance operators require access to the local network in order to reach the final consumers. That is, the potentially competitive market needs to use the monopolized commodity as an input. The purpose of the paper is to develop a normative theory of how to price access. We start by postulating that the regulator has complete information and faces a social cost for public funds associated with the deadweight loss of taxation. The access price must then exceed marginal cost in order to contribute to the reduction of the overall deficit. Equivalently, access can be priced at marginal cost and a tax levied on access. Section 3 studies the impact of the regulator's incomplete information about the monopoly's cost structure on the monopoly's informational rents. Incomplete information may call for a further departure from marginal cost pricing of access. The further distortion, if any, is meant to reduce the monopoly's rent. Section 4 explores conditions under which the observability of the monopoly's subcosts, namely the cost related to the monopolized good and the cost related to the competitive good respectively, improves social welfare.

The analysis is extended in Section 5 to the case where the monopoly does not receive any monetary transfer from the regulator and therefore must cover its cost with its revenue. The access charge is then higher or lower than in the absence of a firm-level budget constraint, depending on whether the monopoly's fixed cost is high or low. Section 6 generalizes our access pricing formula to the case in which the substitute commodity is produced by a competitor with market power rather than by a competitive industry and Section 7 considers the case in which the regulator has no mandate to regulate the competitive segment at all (including the production of the competitive good by the monopoly.)

Sections 6 and 7 emphasize the complexity of regulating a competitive segment with limited instruments. Limitations on regulatory instruments are particularly penalizing when consumers of the competitive commodity can bypass the use of the monopolized



good (local telephone). The determination of the access price must then trade off the conflicting goals of limiting inefficient bypass and of contributing to budget balance (be it the State's or the firm's). Unless the regulator is able to simultaneously tax the long distance market to raise funds to cover the firm's fixed cost and use the access charge to approximate the efficient level of bypass, only a rough balance between these objectives can be achieved. The separation of the regulatory function from the taxation authorities creates serious difficulties in the presence of bypass (Section 8).

Section 9 links the determination of the access charge with the choice of a market structure and with the promotion of entry in the competitive segment. The efficient component pricing rule recently advocated by some economists is assessed in the light of our normative treatment. Section 10 compares the theoretical precepts with two regulatory practices, namely the fully allocated cost method and the Oftel access pricing rule for British Telecom. The Conclusion provides a brief summary of our analysis and discusses avenues of further research.

## 2 The Model

A monopoly operates a network with cost function :

$$C_0 = C_0(\beta, e_0, Q) \quad , \quad C_{0\beta} > 0 \quad , \quad C_{0e_0} < 0 \quad , \quad C_{0Q} > 0, \quad (1)$$

where  $Q$  is the level of network activity,  $\beta$  is a parameter of productivity, and  $e_0$  a level of nonmonetary effort exerted by the firm in operating the network.

$\beta$  is a parameter of adverse selection which is private information of the firm ;  $\beta$  has a c.d.f.  $F(\cdot)$  on  $[\underline{\beta}, \bar{\beta}]$  with a strictly positive density function  $f$ . We make the classical monotone hazard rate assumption ( $\frac{d}{d\beta}[F(\beta)/f(\beta)] \geq 0$ ).

With the network the firm produces a quantity  $q_0$  of a monopolized commodity (local telephone). Let us assume that  $q_0$  units of this commodity require  $q_0$  units of network and let  $S(q_0)$  be consumers' utility for this good, with  $S' > 0$ ,  $S'' < 0$ .

The monopoly produces also a good 1 (long distance telephone) in quantity  $q_1$ . This other production has an imperfect substitute produced in quantity  $q_2$  by a competitive fringe. Let  $V(q_1, q_2)$  be consumers' utility for these two commodities.

In addition to the network input the production of good 1 also generates a cost

$$C_1 = C_1(\beta, e_1, q_1) \quad (2)$$

where  $e_1$  is the effort exerted to reduce the cost of producing good 1. The disutility of

effort is  $\psi(e_0 + e_1)$  with  $\psi' > 0$ ,  $\psi'' > 0$  and  $\psi''' \geq 0$ .

In addition to the network input the production of good 2 generates a cost  $cq_2$  where  $c$  is common knowledge. Because we wish to focus on the network monopoly's incentive to provide access, we need not investigate incentive issues in the fringe.

Total network activity is therefore  $Q = q_0 + q_1 + q_2$ . Let  $a$  denote the access unit charge paid by the competitive fringe to the monopoly. The fringe's level of profit is then :

$$\Pi = p_2 q_2 - c q_2 - a q_2. \quad (3)$$

Under *complete information*, the utilitarian regulator observes prices, quantities, costs, and effort levels. Let  $t$  denote the net transfer received by the monopoly from the regulator. We make the accounting convention that the regulator reimburses costs, receives directly the revenue from the sale of the competitive good and network good to the consumers, and that the firm receives the access charges. This obviously involves no loss of generality. The monopoly's utility level is then :

$$U = t - \psi(e_0 + e_1) + a q_2. \quad (4)$$

The regulator must raise  $t + C_0 + C_1 - p_0 q_0 - p_1 q_1$  with a shadow price of public funds  $1 + \lambda$  (where  $\lambda > 0$  because of distortive taxation). The consumers'/taxpayers' utility is

$$S(q_0) + V(q_1, q_2) - p_0 q_0 - p_1 q_1 - p_2 q_2 - (1 + \lambda)(t + C_0 + C_1 - p_0 q_0 - p_1 q_1). \quad (5)$$

Under complete information an utilitarian regulator would maximise

$$\begin{aligned} (5) + (4) + (3) = & S(q_0) + V(q_1, q_2) - p_0 q_0 - p_1 q_1 - p_2 q_2 - (1 + \lambda)(t + C_0 + C_1 - p_0 q_0 - p_1 q_1) \\ & + [t - \psi(e_0 + e_1) + a q_2] + [p_2 q_2 - c q_2 - a q_2] \end{aligned} \quad (6)$$

under the individual rationality (IR) constraints of the monopoly and the fringe

$$U = t - \psi(e_0 + e_1) + a q_2 \geq 0 \quad (7)$$

$$\Pi = p_2 q_2 - c q_2 - a q_2 \geq 0. \quad (8)$$

Since public funds are costly, the monopoly's IR constraint is binding and social welfare can be rewritten

$$S(q_0) + V(q_1, q_2) + \lambda p_0 q_0 + \lambda p_1 q_1 + \lambda a q_2 - c q_2 - (1 + \lambda) \left( \psi(e_0 + e_1) + C_0(\beta, e_0, q_0 + q_1 + q_2) + C_1(\beta, e_1, q_1) \right). \quad (9)$$

Similarly, raising money through the access charge from the fringe is valuable (see the term  $\lambda a q_2$  in social welfare). The access charge is chosen to saturate the fringe's IR constraint :

$$a = p_2 - c. \quad (10)$$

Substituting (10) in (9), social welfare takes the final form

$$S(q_0) + V(q_1, q_2) + \lambda(p_0 q_0 + p_1 q_1 + p_2 q_2) - (1 + \lambda) \left( \psi(e_0 + e_1) + C_0(\beta, e_0, q_0 + q_1 + q_2) + C_1(\beta, e_1, q_1) + c q_2 \right). \quad (11)$$

Assuming that  $S$  and  $V$  are concave and  $C_0$  and  $C_1$  convex in  $(e_0, e_1, q_1, Q)$ , optimal regulation is characterized by the first-order conditions, that we can write

$$L_0 = \frac{p_0 - C_{0Q}}{p_0} = \frac{\lambda}{1 + \lambda} \frac{1}{\hat{\eta}_0} \quad (12)$$

$$L_1 = \frac{p_1 - C_{0Q} - C_{1q_1}}{p_1} = \frac{\lambda}{1 + \lambda} \frac{1}{\hat{\eta}_1} \quad (13)$$

$$L_2 = \frac{p_2 - C_{0Q} - c}{p_2} = \frac{\lambda}{1 + \lambda} \frac{1}{\hat{\eta}_2} \quad (14)$$

where  $\hat{\eta}_0, \hat{\eta}_1, \hat{\eta}_2$  are (respectively) the price superelasticities of goods 0, 1, 2, and

$$\psi'(e_0 + e_1) = -C_{0e_0} = -C_{1e_1}. \quad (15)$$

Straightforward computations show that  $\hat{\eta}_0 = \eta_0$  and

$$\hat{\eta}_1 = \eta_1 \frac{(\eta_1 \eta_2 - \eta_{12} \eta_{21})}{\eta_1 \eta_2 + \eta_1 \eta_{12}} < \eta_1 \quad (16)$$

$$\hat{\eta}_2 = \eta_2 \frac{(\eta_1 \eta_2 - \eta_{12} \eta_{21})}{\eta_1 \eta_2 + \eta_2 \eta_{21}} < \eta_2 \quad (17)$$

where  $\eta_i = -\frac{\partial q_i}{\partial p_i} \frac{p_i}{q_i}$ .

$$\eta_{ij} = \frac{\partial q_i}{\partial p_j} \frac{p_j}{q_i} \quad j \neq i, \quad j = 1, 2, \quad i = 1, 2.$$

The benchmark pricing rules are Ramsey pricing rules because  $\lambda > 0$ . The access charge can be rewritten

$$a = C_{0Q} + \frac{\lambda}{1 + \lambda} \frac{p_2}{\hat{\eta}_2}. \quad (18)$$

*Access is priced above marginal cost because deficits are socially costly.* The term  $\frac{\lambda p_2}{(1 + \lambda) \hat{\eta}_2}$  can be viewed as a tax used to raise money. It is high when the elasticity of good 2 is low or when the social cost of funds is high. In the next section we address various issues raised by asymmetric information.

If the regulator has two instruments, the access price and a tax on good 2,  $\tau_2$ , equation (14) can be rewritten  $a + \tau_2 = C_{0Q} + \frac{\lambda}{1 + \lambda} \frac{p_2}{\hat{\eta}_2}$  and  $a$  can obviously be taken equal to the marginal cost of access. Furthermore, it is clear that if the regulator had access to a nondistortive (lump sum) tax, marginal cost pricing of commodities and access would be optimal.

### 3 Access Pricing and Incentives

The first issue we consider is the extent to which asymmetric information affects access pricing. Let us consider the case where accounting rules enable the regulator to observe separately  $C_0$  and  $C_1$ .

Let  $E_0(\beta, C_0, Q)$  denote the solution in  $e_0$  of  $C_0 = C_0(\beta, e_0, Q)$  and let  $E_1(\beta, C_1, q_1)$  the solution in  $e_1$  of  $C_1 = C_1(\beta, e_1, q_1)$ . The firm's utility can be written

$$t + a q_2 - \psi(E_0(\beta, C_0, Q) + E_1(\beta, C_1, q_1)). \quad (19)$$

Appealing to the revelation principle we consider a revelation mechanism

$$\{t(\tilde{\beta}), C_0(\tilde{\beta}), C_1(\tilde{\beta}), q_1(\tilde{\beta}), q_2(\tilde{\beta}), Q(\tilde{\beta}), a(\tilde{\beta})\} \quad (20)$$

which specifies the transfer received, the sub-costs to be realized, the quantities to be produced, and the access price to be received if the firm announces a characteristic  $\tilde{\beta}$ .

Neglecting momentarily second-order conditions, and using the rent variable

$$U(\beta) = t(\beta) + a(\beta) q_2(\beta) - \psi(E_0(\beta, C_0(\beta), Q(\beta)) + E_1(\beta, C_1(\beta), q_1(\beta))),$$



the incentive constraint of the monopoly can be written

$$\dot{U}(\beta) = -\psi'(e_0 + e_1) \left( \frac{\partial E_0}{\partial \beta} + \frac{\partial E_1}{\partial \beta} \right). \quad (21)$$

Since  $\frac{\partial E_0}{\partial \beta} > 0$  and  $\frac{\partial E_1}{\partial \beta} > 0$ , the rent is decreasing in  $\beta$  and the individual rationality (IR) constraint ( $U(\beta) \geq 0$  for all  $\beta$ ) boils down to

$$U(\bar{\beta}) \geq 0. \quad (22)$$

Optimal regulation results from the maximization, subject to (21) and (22), of :

$$\begin{aligned} & \int_{\underline{\beta}}^{\bar{\beta}} \left\{ S(q_0(p_0(\beta))) + V(q_1(p_1(\beta), p_2(\beta)), q_2(p_1(\beta), p_2(\beta))) \right. \\ & \quad \left. + \lambda [p_0(\beta)q_0(p_0(\beta)) + p_1(\beta)q_1(p_1(\beta), p_2(\beta)) + p_2(\beta)q_2(p_1(\beta), p_2(\beta))] \right. \\ & \quad \left. - (1 + \lambda) [\psi(e_0(\beta) + e_1(\beta)) + C_0(\beta, e_0(\beta), q_0(p_0(\beta)) + q_1(p_1(\beta), p_2(\beta)) + q_2(p_1(\beta), p_2(\beta))) \right. \\ & \quad \left. + C_1(\beta, e_1(\beta), q_1(p_1(\beta), p_2(\beta))) + cq_2(p_1(\beta), p_2(\beta))] - \lambda U(\beta) \right\} dF(\beta). \end{aligned} \quad (23)$$

We obtain (see appendix 1).

$$\frac{p_0 - C_{0Q}}{p_0} = \frac{\lambda}{1 + \lambda} \frac{1}{\hat{\eta}_0} + \frac{\lambda}{1 + \lambda} \frac{F}{f} \frac{\psi'}{p_0} \frac{\partial}{\partial Q} \left( - \frac{C_{0\beta}(\beta, e_0, Q)}{C_{0e_0}(\beta, e_0, Q)} \right) \quad (24)$$

$$\begin{aligned} & \frac{p_1 - C_{0Q} - C_{1q_1}}{p_1} = \frac{\lambda}{1 + \lambda} \frac{1}{\hat{\eta}_1} \\ & + \frac{\lambda}{1 + \lambda} \frac{F}{f} \frac{\psi'}{p_1} \left( \frac{\partial}{\partial Q} \left\{ - \frac{C_{0\beta}(\beta, e_0, Q)}{C_{0e_0}(\beta, e_0, Q)} \right\} + \frac{\partial}{\partial q_1} \left\{ - \frac{C_{1\beta}(\beta, e_1, q_1)}{C_{1e_1}(\beta, e_1, q_1)} \right\} \right) \end{aligned} \quad (25)$$

$$\frac{p_2 - C_{0Q} - c}{p_2} = \frac{\lambda}{1 + \lambda} \frac{1}{\hat{\eta}_2} + \frac{\lambda}{1 + \lambda} \frac{F}{f} \frac{\psi'}{p_2} \frac{\partial}{\partial Q} \left\{ - \frac{C_{0\beta}(\beta, e_0, Q)}{C_{0e_0}(\beta, e_0, Q)} \right\}. \quad (26)$$

All prices are modified by an incentive correction related to the sub-cost function  $C_0$  and  $p_1$  has an additional correction associated with  $C_1$ .

The access charge can now be written

$$a = C_{0Q} + \frac{\lambda}{1 + \lambda} \frac{p_2}{\hat{\eta}_2} + \frac{\lambda}{1 + \lambda} \frac{F}{f} \psi' \frac{\partial}{\partial Q} \left\{ - \frac{C_{0\beta}}{C_{0e_0}} \right\}. \quad (27)$$

Let us analyse the new term, which is due to incentive constraints. First, we observe that there is no correction for the most efficient type as  $F(\underline{\beta}) = 0$ . Second,  $\frac{\partial E_0}{\partial \beta} =$

$-C_{0\beta}/C_{0e_0}$  is the rate at which the firm must substitute effort and productivity to keep the same level of cost for the network activity. From (21), we see that it is a crucial term to determine the firm's rent. The regulator wants to limit this costly rent. If  $\frac{\partial}{\partial Q}(\frac{\partial E_0}{\partial \beta})$  is positive, he raises the access price and *therefore reduces quantities to extract rent and conversely if it is negative*. The effect of incentives on the access charge thus depends on fine characteristics of the cost function.

Access charges must be increased (decreased) for incentive reasons if the cost function of the network  $C_0$  is such that

$$\overbrace{C_{0e_0Q}C_{0\beta}}^{?} - \overbrace{C_{0\beta Q}C_{0e_0}}^{>0} > (<) 0.$$

With the "Spence-Mirrlees" condition on cost,  $C_{0\beta Q} > 0$  and our previous assumptions, a clear positive sign is obtained if effort increases the marginal cost. However, if, as is more reasonable, effort decreases the marginal cost, the effect is ambiguous.

Intuitively, production must be decreased (increased) if an increase in the level of  $Q$  increases (decreases) the "ability" of the firm to lie about its characteristics (in elasticity terms, if the quantity elasticity of the marginal effect on cost of the productivity characteristics  $\beta$  is larger (smaller) than the quantity elasticity of the marginal effect on cost of effort).

From Leontief's theorem we know<sup>5</sup> in addition that the incentive effect on the access charge (and on good 1) disappears iff there exists a function  $\xi$  such that

$$C_0 = C_0(\xi(\beta, e_0), Q). \quad (28)$$

If furthermore  $C_1$  is separable in  $(\beta, e_1)$  on the one hand, and  $q_1$  on the other hand, there is no incentive distortion on any commodity : the dichotomy between pricing rules (unaffected) and the cost reimbursement rules (see below) holds.

The access pricing rule can then be rewritten :

$$a = C_{0Q} + \frac{\lambda}{1 + \lambda} \frac{p_2}{\hat{\eta}_2}. \quad (29)$$

Note that

$$\frac{p_1 - C_{0Q} - C_{1q_1}}{p_1} = \left[ \frac{\hat{\eta}_2}{\hat{\eta}_1} \right] \left[ \frac{a - C_{0Q}}{p_2} \right].$$

It is also possible to find conditions on the cost function that yield nonambiguous conclusions for the incentive correction when it exists. For instance, the following class of cost functions leads to a well defined sign of the incentive correction in the access charge :

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<sup>5</sup>From Leontief's theorem we know that  $\frac{C_{0\beta}}{C_{0e_0}}$  independent of  $Q$  is equivalent to the existence of a function  $\xi(\cdot)$  such that (29) holds (see Laffont-Tirole (1990a)).

$$C_0 = \beta Q^d - e_0^c Q^b.$$

If  $d > b$  ( $d < b$ ), the incentive correction is positive (negative).

Let us now consider the effort levels (in the case of dichotomy) obtained by maximizing (23) under (21) and (22) with respect to  $(e_0, e_1)$ . We obtain

$$\begin{aligned} \psi'(e_0 + e_1) &= -C_{0e_0} \\ -\frac{2\lambda F}{(1+\lambda)f} &\left[ \psi'(e_0 + e_1) \left( \frac{\partial E_0}{\partial \beta} + \frac{\partial E_1}{\partial \beta} \right) + \psi'(e_0 + e_1) \frac{\partial^2 E_0}{\partial \beta \partial C_0} C_{0e_0} \right] \end{aligned} \quad (30)$$

$$\begin{aligned} \psi'(e_0 + e_1) &= -C_{1e_1} \\ -\frac{2\lambda F}{(1+\lambda)f} &\left[ \psi'(e_0 + e_1) \left( \frac{\partial E_0}{\partial \beta} + \frac{\partial E_1}{\partial \beta} \right) + \psi'(e_0 + e_1) \frac{\partial^2 E_1}{\partial \beta \partial C_1} C_{1e_1} \right]. \end{aligned} \quad (31)$$

Equations (30) and (31) can be interpreted as cost reimbursement rules. If the conditions underlying the irrelevance of subcost observability (see Section 4) hold, one can obtain sufficient conditions for the optimal contract to be implementable through a single menu of linear sharing rules  $t = A - B(C_0 + C_1)$  where  $B = \psi' \frac{\partial E_0}{\partial C_0} = \psi' \frac{\partial E_1}{\partial C_1}$  is the share of the total cost which is reimbursed. From (30), (31), and  $F(\beta) = 0$ , we see that the most efficient type receives from the regulator a lump sum transfer (fixed price contract) and equates its disutility of effort to the marginal cost reduction as under complete information.

## REMARKS

1. - The most relevant cost functions seem to be such that higher production levels call for higher effort levels and therefore induce higher rents. Incentive considerations raise marginal costs and therefore indirectly call for a general reduction of quantities. Access prices are increased for the competitor but simultaneously the monopoly is penalized by high prices for its own products. Because access costs are the same for the monopolist's own product and the competitor's, the current model may understate the incentives for misrepresentation. The firm cannot claim simultaneously high access costs and high efficiency of its own products (see Laffont-Tirole (1993, Chapter 5) for an example where this link is suppressed).

2. - If the fringe is not regulated but is competitive, the same results obtain ;  $p_2 = a + c$  is then ensured by competition rather than by the existence of a shadow cost of public funds.

3. - Let us consider implementation of the optimal revelation mechanism with the following specification :

$$C_0 = H_0(\beta - e_0)(q_0 + q_1 + q_2) \quad (32)$$

$$C_1 = H_1(\beta - e_1)q_1. \quad (33)$$

The objective function of the monopoly can be rewritten

$$\hat{t}(\beta) - \psi\left(2\beta - H_0^{-1}\left(\frac{C_0}{q_0 + q_1 + q_2}\right) - H_1^{-1}\left(\frac{C_1}{q_1}\right)\right) \quad (34)$$

where  $\hat{t}$  includes now the access charge. The second-order condition of incentive compatibility is

$$\frac{d}{d\beta}\left\{H_0^{-1}\left(\frac{C_0}{q_0 + q_1 + q_2}\right) + H_1^{-1}\left(\frac{C_1}{q_1}\right)\right\} \geq 0 \quad (35)$$

and the revelation mechanism is equivalent to a nonlinear reward function :

$$T\left(H_0^{-1}\left(\frac{C_0}{q_0 + q_1 + q_2}\right) + H_1^{-1}\left(\frac{C_1}{q_1}\right)\right). \quad (36)$$

The firm is now free to choose its access charge but it knows that it will affect the price and the quantity of good 1 (as well as good 2). It can be advised to choose the access price defined in formula (27) since it is indifferent among all prices leading to the same weighted cost, which determines its rent.

An alternative implementation is obtained by writing the transfer as a function of costs  $C_0$ ,  $C_1$ , production levels  $q_0$ ,  $q_1$  and access charge  $a$  it picks. For this purpose consider the conditional demand function of good 2,  $q_2 = D_2(q_1, c + a)$  and substitute in (36). It is easy to check that the transfer received by the firm is decreasing in the access charge it chooses.

4. - Suppose that the monopoly selects the level of some unobservable action  $i$  (an investment for example) which raises the cost for the network, but decreases the cost of its service (good 1) :

$$C_0 = \frac{i^2}{2}Q + C_0(\beta, e_0, Q)$$

$$C_1 = -iq_1 + C_1(\beta, e_1, q_1).$$



Under complete information the optimal investment level, which minimizes total cost, is  $i = q_1/Q$ . Under incomplete information, the moral hazard constraint on investment is

$$-i \frac{\partial E_0}{\partial C_0} Q + \frac{\partial E_1}{\partial C_1} q_1 = 0$$

or

$$i = \frac{q_1}{Q} \frac{\partial E_1 / \partial C_1}{\partial E_0 / \partial C_0}.$$

To decrease the rent of asymmetric information the regulator, who observes both  $C_0$  and  $C_1$ , may use cost reimbursement rules with different rates for sharing overruns (implying  $\frac{\partial C_0}{\partial E_0} \neq \frac{\partial C_1}{\partial E_1}$ ) and this may lead to a distortion of investment (which would not exist if the regulator were not observing separately sub-costs  $C_0$  and  $C_1$  as in the next section).

If  $i$  is too low (too high), the access price is increased (decreased) in order to favor good 1 (favor good 2) and to foster an increase (decrease) of investment. A low access charge may be viewed by the firm as a nonrecognition of its investment costs in the network. It is indeed low to decrease those types of investment which may be excessive.

## 4 On Sub-Cost Observability

Section 3 derived the optimal regulation under the assumption that the regulator could audit the costs of the regulated monopoly well enough to separate the cost of the network from the cost of the commodity produced in the competitive sector.

Using  $C_k$  instead of  $e_k$  in the optimization leading to (30) and (31), we have

$$\begin{aligned} & \left[ (1 + \lambda) \psi' + \lambda F \psi'' \frac{\partial(E_0 + E_1)}{\partial \beta} \right] \left[ - \frac{\partial E_k}{\partial C_k} \right] \\ &= (1 + \lambda) f + \lambda F \psi' \frac{\partial^2 E_k}{\partial C_k \partial \beta}, \quad k = 0, 1. \end{aligned} \tag{37}$$

A high-powered scheme on activity  $k$  corresponds to a case where the transfer received by the firm is highly dependent on cost  $C_k$ , i.e. where  $(-\frac{\partial E_k}{\partial C_k})$  is high. Indeed if  $\frac{\partial E_k}{\partial C_k}$  is high in absolute value, marginal cost reductions on activity  $k$  per unit of effort are small and therefore the firm is already induced to exert a large effort on activity  $k$ . Also, note for further reference that if the cross terms  $\frac{\partial^2 E_k}{\partial C_k \partial \beta}$  vanish the terms  $(-\frac{\partial E_k}{\partial C_k})$  are equal.

If sub-costs are not observable, the firm minimizes its effort for any cost target. It chooses  $\{e_k\}$  or, equivalently  $\{C_k\}$ , so as to minimize  $\sum_{k=0}^1 E_k(\beta, C_k, Q_k)$  subject to  $C_0 +$

$C_1 = C$  (with the convention  $Q_0 = Q, Q_1 = q_1$ ). Hence, in the absence of sub-cost observation the marginal effort levels to reduce sub-costs ( $-\frac{\partial E_k}{\partial C_k}$ ) are equalized<sup>6</sup>.

Whether the regulator wants to give differentiated incentives on different activities in the case of sub-cost observability hinges on the cross-partial derivatives  $\frac{\partial^2 E_k}{\partial C_k \partial \beta}$  (note that this expression is in absolute value proportional to  $\frac{\partial^2 E_k}{\partial e_k \partial \beta}$ ).  $\frac{\partial E_k}{\partial \beta}$  measures the rate at which the firm can substitute  $\beta$  and effort in activity  $k$  (see Section 3) and it is the main determinant of the firm's rent. If the level of cost or the level of effort does not affect this rate, there is no point in using sub-cost observability to try to affect the firm's allocation of effort to minimize cost in order to reduce rents.

More formally, let us assume that the minimization of total effort  $\sum_{k=0}^1 E_k(\beta, C_k, Q_k)$  subject to  $C_0 + C_1 = C$  has a unique solution  $e_k = \hat{E}_k(\beta, C, Q_0, Q_1)$ , which is the unique solution to the system  $\{ \frac{\partial E_k}{\partial C_k} = \frac{\partial E_\ell}{\partial C_\ell}$  for all  $(k, \ell)$  and  $C_0 + C_1 = C \}$ .

Solving for the optimal regulation we obtain (see appendix 2) :

*Under sub-cost observability :*

- i) *the firm faces higher incentives on those activities for which low costs reduce rents ( $\frac{\partial^2 E_k}{\partial C_k \partial \beta}$  large).*
- ii) *the firm faces uniform incentives on all activities and therefore sub-cost observability is useless if  $\partial^2 E_k / \partial C_k \partial \beta = K$  for all  $k$  and some constant  $K$ .*

This result tells us that the incentive constraints of the firm are identical with or without sub-cost observability when  $\partial^2 E_k / \partial C_k \partial \beta = \text{constant}$  for  $k = 0, 1$ . Consequently the optimal pricing rules and in particular the optimal access pricing is unaffected in this case by sub-cost unobservability.

A characterization of the subclass of cost functions satisfying the sub-cost-irrelevance property with  $K = 0$  is easily obtained. For all  $k$ , the subcost functions must be such that :

$$\frac{\partial^2 E_k}{\partial C_k \partial \beta} = 0 \text{ for } k = 0, 1 \Leftrightarrow e_k = E_k(\beta, C_k, Q_k) = H_k(\beta, Q_k) + G_k(C_k, Q_k)$$

for some functions  $H_k$  and  $G_k$  for  $k = 0, 1$  and, since  $C_{e_k} < 0$ ,  $C_k = \hat{C}_k(Q_k, H_k(\beta, Q_k) - e_k)$ .

On the other hand, recall that the dichotomy property holds when :

$$C_k = C_k(\xi_k(\beta, e_k), Q_k) \quad \text{for } k = 0, 1.$$

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<sup>6</sup>Then, the theory of access pricing is similar to the one of Section 3 but for marginal costs evaluated at the effort levels induced by the incentive scheme defined when sub-costs are not observable.

These related conditions are different. The dichotomy requires

$$\frac{d}{dQ_k} \left( \frac{\partial E_k}{\partial \beta} \right) = \frac{\partial^2 E_k}{\partial C_k \partial \beta} \frac{\partial C_k}{\partial Q_k} + \frac{\partial^2 E_k}{\partial Q_k \partial \beta} = 0,$$

while the irrelevance of sub-cost observation requires

$$\frac{\partial^2 E_k}{\partial C_k \partial \beta} = K \text{ for all } k.$$

The class of cost functions

$$C_k(\beta, e_k, q_k) = \beta q_k^{d_k} - e_k q_k^{b_k}$$

satisfies the sub-cost irrelevance property for any  $d_k, b_k$  (with  $C_{kq_k} > 0$ ) and the dichotomy property for  $d_k = b_k$  only.

On the other hand,  $C_k(\beta, e_k, q_k) = \frac{\beta q_k}{e_k}$  satisfies the dichotomy property but not the sub-cost irrelevance property.

## 5 Optimal Access Pricing in the Absence of Government Transfers

We now assume that the regulator is prohibited from transferring money to the firm. Let us for example consider a constant returns to scale case where the dichotomy and the irrelevance of subcost observability hold :

$$C_0 = H_0(\beta - e_0)(q_0 + q_1 + q_2)$$

$$C_1 = H_1(\beta - e_1)q_1$$

and let  $c_0 \equiv H_0(\beta - e_0)$ ,  $c_1 \equiv H_1(\beta - e_1)$  (we omit the fixed cost for notational simplicity).

Under symmetric information, the budget constraint of the monopoly is

$$p_0 q_0(p_0) + p_1 q_1(p_1, a + c) + a q_2(p_1, a + c)$$

$$- c_0(q_0(p_0) + q_1(p_1, a + c) + q_2(p_1, a + c)) - c_1 q_1(p_1, a + c) - t \geq 0, \quad (38)$$

where  $t$  must now be interpreted as the firm's manager's compensation and is paid from consumer charges (so  $U = t - \psi(e_0 + e_1)$ ).

The regulator wishes to maximize, for each value of  $c_0, c_1, t$ , social welfare :

$$S(q_0(p_0)) + V(q_1(p_1, a+c), q_2(p_1, a+c)) - p_0 q_0(p_0) - p_1 q_1(p_1, a+c) - (a+c) q_2(p_1, a+c) + U$$

subject to constraint (38) (where  $U$  is the firm's welfare).

Under asymmetric information, the firm's rent, namely

$$U(\beta) = t(\beta) - \psi(e_0(\beta) + e_1(\beta)),$$

leads to the incentive and individual rationality constraints :

$$\dot{U}(\beta) = -\psi'(e_0(\beta) + e_1(\beta)) \quad (39)$$

$$U(\bar{\beta}) \geq 0. \quad (40)$$

The budget constraint is, for each  $\beta$  :

$$\begin{aligned} & p_0(\beta) q_0(p_0(\beta)) + p_1(\beta) q_1(p_1(\beta), a(\beta) + c) + a(\beta) q_2(p_1(\beta), a(\beta) + c) \\ & - c_0(q_0(p_0(\beta)) + q_1(p_1(\beta), a(\beta) + c) + q_2(p_1(\beta), a(\beta) + c)) \\ & - c_1 q_1(p_1(\beta), a(\beta) + c) = U(\beta) + \psi(e_0(\beta) + e_1(\beta)). \end{aligned} \quad (41)$$

The regulator's optimization program is :

$$\begin{aligned} \max_{\underline{\beta}} \int_{\underline{\beta}}^{\bar{\beta}} & \left[ S(q_0(p_0(\beta))) + V(q_1(p_1(\beta), a(\beta) + c), q_2(p_1(\beta), a(\beta) + c)) - p_0(\beta) q_0(p_0(\beta)) \right. \\ & \left. - p_1(\beta) q_1(p_1(\beta), a(\beta) + c) - p_2(\beta) q_2(p_1(\beta), a(\beta) + c) + U(\beta) \right] dF(\beta) \\ \text{s.t. } & (39), (40), (41). \end{aligned}$$

Let  $\mu(\beta)$  be the Pontryagin multiplier of the state variable  $U$  and  $(1 + \bar{\lambda}(\beta))f(\beta)$  the multiplier of the budget constraint. Optimizing over prices *we obtain the same equations as in Section 2 with  $\bar{\lambda}(\beta)$  replacing  $\lambda$* . Using the Pontryagin principle and the transversality condition  $\mu(\underline{\beta}) = 0$ , we have



$$\mu(\beta) = \int_{\underline{\beta}}^{\beta} \tilde{\lambda}(\tilde{\beta}) f(\tilde{\beta}) d\tilde{\beta}.$$

Optimizing over effort levels we get finally :

$$\psi'(e_0(\beta) + e_1(\beta)) = H'_0(q_0(\beta) + q_1(\beta) + q_2(\beta)) - \frac{\int_{\underline{\beta}}^{\beta} \tilde{\lambda}(\tilde{\beta}) f(\tilde{\beta}) d\tilde{\beta}}{(1 + \tilde{\lambda}(\beta)) f(\beta)} \psi''(e_0(\beta) + e_1(\beta))$$

and

$$\psi'(e_0(\beta) + e_1(\beta)) = H'_1 q_1(\beta) - \frac{\int_{\underline{\beta}}^{\beta} \tilde{\lambda}(\tilde{\beta}) f(\tilde{\beta}) d\tilde{\beta}}{(1 + \tilde{\lambda}(\beta)) f(\beta)} \psi''(e_0(\beta) + e_1(\beta)).$$

When the dichotomy does not hold, formulae similar to those of Section 3 are obtained with  $\frac{\lambda F(\beta)}{(1+\lambda)f(\beta)}$  replaced by

$$\frac{\int_{\underline{\beta}}^{\beta} \tilde{\lambda}(\tilde{\beta}) f(\tilde{\beta}) d\tilde{\beta}}{(1 + \tilde{\lambda}(\beta)) f(\beta)}.$$

In particular the access pricing equation becomes

$$a = C_{0Q} + \frac{\tilde{\lambda}(\beta)}{1 + \tilde{\lambda}(\beta)} \frac{p_2}{\hat{\eta}_2} + \frac{\int_{\underline{\beta}}^{\beta} \tilde{\lambda}(\tilde{\beta}) f(\tilde{\beta}) d\tilde{\beta}}{(1 + \tilde{\lambda}(\beta)) f(\beta)} \psi' \frac{\partial}{\partial Q} \left\{ \frac{-C_{0\beta}}{C_{0e_0}} \right\} \quad (42)$$

Under the assumption of dichotomy, the ratios of Lerner indices are unaffected by the lack of transfers, but the whole price structure is shifted upwards or downwards depending on how binding the budget constraint is. Consequently, *the access charge is higher (lower) than in the absence of budget constraint when the fixed cost is high (low).*

## 6 Access Pricing and Competitor with Market Power

Competition in the markets using the network as an input is often imperfect, as is the case for competition in long distance telecommunications, or competition in the generation of electricity in England. To account for this market power we pursue the balanced-budget analysis of Section 5 with the same technology (guaranteeing the dichotomy and the irrelevance of sub-cost observation).

If the competitor has market power and is unregulated, the maximization of expected welfare must be carried under the constraint that the competitor chooses price so as to maximize its profit :

$$q_2(p_1, p_2) + p_2 \frac{\partial q_2}{\partial p_2}(p_1, p_2) - (c + a) \frac{\partial q_2}{\partial p_2}(p_1, p_2) = 0.^7 \quad (43)$$

From the first-order condition of this maximization program (see appendix 3) we obtain the access pricing rule :

$$a = C_{0Q} - \frac{p_2}{\eta_2} + \frac{\tilde{\lambda}(\beta)p_2}{1 + \tilde{\lambda}(\beta)} \left\{ \frac{1}{\eta_2^*} - \frac{p_2\eta_{11}\frac{\partial}{\partial p_2}\left(\frac{1}{\eta_2}\right) + p_1\eta_{12}\frac{\partial}{\partial p_1}\left(\frac{1}{\eta_2}\right)}{\eta_{11}\eta_2 - \eta_{12}\eta_{21}} \right\} \quad (44)$$

with

$$\eta_2^* = \eta_2 \frac{\eta_{11}\eta_2 - \eta_{12}\eta_{21}}{2\eta_{11}\eta_2 - \eta_{11} + \eta_{21}\eta_2 - \eta_{21}\eta_{12}}.$$

Equation (44) is complex, but can be given a natural interpretation. The two terms in the derivatives of the elasticity of demand for good 2 describe the effects of changes in  $p_1$  and  $p_2$  on the *mark up* or monopoly power  $1/(1 - \frac{1}{\eta_2})$ . More interesting is the superelasticity  $\eta_2^*$  corrected for market power. It exceeds the regular superelasticity  $\hat{\eta}_2$  if and only if  $\eta_{21}\eta_{12} > \eta_{11}(\eta_2 - 1)$ . In the *independent* demand case ( $\eta_{21} = \eta_{12} = 0$ ),  $\eta_2^* < \hat{\eta}_2$  and therefore (assuming a constant elasticity demand for good 2), the need to balance the budget (reflected in  $\tilde{\lambda} > 0$ ) calls for a higher final price than in the absence of monopoly power. To explain this, notice that the monopoly power amounts to a social loss equal to  $\tilde{\lambda}$  times the monopoly's profit  $p_2q_2/\eta_2$ , once the *standard subsidy*  $p_2q_2/\eta_2$  (see (44)) is made to offset the monopoly distortion. Because marginal revenue is decreasing, *an increase in  $p_2$  (that is, in the access price) reduces this monopoly profit*. Hence  $\eta_2^* < \hat{\eta}_2$ . However, with *dependent* demands, there is a second effect (described by the term  $\eta_{21}\eta_{12}$  in the expression of  $\eta_2^*$ ) :  $p_1$  is reduced to lower the monopoly profit  $p_2q_2/\eta_2$ . To *rebalance the consumers' choice* between the two substitutes,  $p_2$  must also be reduced, and so must be the access charge.

If the social welfare function did not include the competitor's profit (an extreme way of depicting the redistribution problem), similar calculations would yield :

$$a = C_{0Q} + \frac{\tilde{\lambda}(\beta)}{1 + \tilde{\lambda}(\beta)} \frac{p_2}{\hat{\eta}_2} - \frac{p_2\eta_{11}\frac{\partial}{\partial p_2}\left(\frac{1}{\eta_2}\right) + p_1\eta_{12}\frac{\partial}{\partial p_1}\left(\frac{1}{\eta_2}\right) + \frac{\eta_1}{\eta_2}}{\eta_{11}\eta_2 - \eta_{12}\eta_{21}} p_2.$$

<sup>7</sup>An alternative formula would be obtained if a nonlinear access pricing rule was used. See the conclusion.

## 7 Restricted Regulation

In practice, the regulator may let (or be instructed to let) the natural monopoly select his pricing behavior in the “competitive market” and regulate only the price of the monopolized good and the access price to the network. For instance, France Télécom is free to set its charges on value added services. This section studies the design and consequences of such restricted regulation under simplifying assumptions. We come back to the case of a competitive fringe. We consider the class of regulatory mechanisms that associate with an announcement  $\hat{p}$  prices  $p_0(\hat{p})$  and  $a(\hat{p})$ . The managerial compensation  $t(\hat{p})$  is then defined as a residual by the balanced budget constraint, once efforts  $e_0$  and  $e_1$  and price  $p_1$  have been chosen.

The analysis proceeds as in Section 5 except that the monopoly has an additional moral hazard variable  $p_1$  which leads to the first-order equation, that the monopoly’s marginal profit be equal to zero :

$$MP = q_1 + \frac{\partial q_1}{\partial p_1}[p_1 - c_0 - c_1] + \frac{\partial q_2}{\partial p_1}[p_2 - c_0 - c] = 0.$$

we make the following assumptions :

Concavity of profit :  $\frac{dMP}{dp_1} < 0$

Generalized strategic complements :  $\frac{dMP}{dp_2} > 0$ .

The standard condition for strategic complementarity is

$$d\left[q_1 + \frac{\partial q_1}{\partial p_1}[p_1 - c_0 - c_1]\right]/dp_2 > 0.$$

Here, the monopolist also receives income from giving access. A sufficient condition for generalized strategic complementarity is strategic complementarity plus the condition that the marginal profit on access increases with  $p_2$  (condition that holds with linear demand and constant returns to scale).

Appendix 6 shows that for a given shadow price, *the access price is lowered relative to the unrestricted control case, in order to reduce the monopoly price through (generalized) strategic complementarity for a given  $p_1$* . On the other hand, monopoly power on good 1 raises  $p_1$ . To rebalance the consumers’ choice between the two goods,  $p_2$  must be raised. The effect of restricted regulation on the access price is therefore ambiguous in general. However, *in the case of linear demand, for the optimal  $p_1$ , the access pricing formula remains the formula of full regulation*.



## 8 Access Pricing and Bypass

A practical problem encountered in the telecommunications sector is the following : giving access to the network is useful as it expands the space of commodities offered and may decrease the rents of asymmetric information by shrinking the incumbent's scale of operation. More directly it may also be useful for yardstick reasons if the technology of the incumbent is correlated with the monopolist's technology. However, in telecommunications for example giving access to the network to long distance operators leads to the possibility of bypass of the local loop by large consumers to connect with these operators.

So far we had no need to distinguish between producer prices and consumer prices of long distance, and we could assume without loss of generality that access pricing was the only tool used to oblige the competitor to participate in the fixed costs of the local loop. However, when bypass is feasible high access prices required for funding these costs induce inefficient bypass.

Nonlinear prices for local telecommunication may help mitigate this dilemma<sup>8</sup>. But, there is a much more efficient way of dealing with this problem which amounts to disconnecting consumer prices and producer prices<sup>9</sup>.

The monopolist has marginal cost  $c_0$  for the local network and  $c_1$  for long distance. The fringe has marginal cost  $c_2$ <sup>10</sup>. Consider the case of a continuum  $[0, 1]$  of consumers with identical preferences  $V(q_1, q_2)$ . Let  $\hat{V}(p_1, p_2)$  be the indirect utility function. The bypass technology is characterized by a fixed cost  $\theta \in [0, \infty)$  and a (low) marginal cost  $b$ .  $b$  is the same for everyone,  $\theta$  is distributed according to a c.d.f.  $G(\theta)$ .

Let  $\tau_2$  be a tax on good 2. With a competitive fringe the price of good 2 is then :

$$p_2 = a + c_2 + \tau_2 .$$

The price of good 2 charged to those who bypass the local loop is

$$p_2 - a = c_2 + \tau_2 .$$

The marginal price of good 2 obtained through bypass is de facto

$$\tilde{p}_2 = p_2 + b - a$$

given that consumer  $\theta$  has paid a fixed cost  $\theta$ .

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<sup>8</sup>See Laffont-Tirole (1990b) for a different example of the use of nonlinear prices to fight bypass.

<sup>9</sup>However, this option is rarely available to regulators.

<sup>10</sup>The dichotomy property is assumed so that we can study pricing issues independently of incentives issues.

For given prices  $(p_1, p_2, a)$  consumers who bypass are those with  $\theta$  lower than  $\theta^*$  defined by

$$\hat{V}(p_1, p_2) = -\theta^* + \hat{V}(p_1, p_2 + b - a).$$

Leaving aside good 0, optimal pricing and access pricing are then the solution of :

$$\begin{aligned} \text{Max} \int_{\underline{\theta}}^{\theta^*} & \left[ \hat{V}(p_1, p_2 + b - a) - \theta + (1 + \lambda) \left[ (p_1 - c_1 - c_0)q_1(a) + (p_2 - a - c_2)q_2(a) \right] \right] dG(\theta) \\ & + \int_{\theta^*}^{\bar{\theta}} \left[ \hat{V}(p_1, p_2) + (1 + \lambda) \left[ (p_1 - c_1 - c_0)q_1 + (p_2 - c_2 - c_0)q_2 \right] \right] dG(\theta) \end{aligned} \quad (45)$$

where for ease of notation

$$\begin{aligned} q_1(a) &= q_1(p_1, p_2 + b - a) & q_2(a) &= q_2(p_1, p_2 + b - a) \\ q_1 &= q_1(p_1, p_2) & q_2 &= q_2(p_1, p_2). \end{aligned}$$

Let  $\eta_1(a), \eta_2(a), \eta_{12}(a), \eta_{21}(a), R_1(a), R_2(a)$  be the elasticities and revenues associated with the demand functions  $q_1(a), q_2(a)$  and the prices  $p_1, p_2$  and let  $\eta_1, \eta_2, \eta_{12}, \eta_{21}, R_1, R_2$  the elasticities and revenues associated with the demand functions  $q_1, q_2$ .

$$\text{Let } \Delta R = (p_1 - c_1 - c_0)(q_1(a) - q_1) + (p_2 - a - c_2)q_2(a) - (p_2 - c_2 - c_0)q_2.$$

$\Delta R$  is an income effect. It is the loss of profits (including access charge and tax on good 2) when a consumer switches to bypass. If we think of the difference between prices and marginal costs as taxes for raising funds, it is the difference of “collected taxes” when the switch occurs.  $\Delta R < 0$  means that less taxes are collected when the switch occurs. This should favor a lower access price than when no switch occurs (see equation A5.3 in appendix 5).

It seems difficult to predict the sign of  $\Delta R$ . So we will give the complete results only for  $\Delta R \approx 0$ <sup>11</sup>. Then, from appendix 5 we have :

$$\frac{p_1 - c_1 - c_0}{p_1} = \frac{\lambda}{1 + \lambda} \frac{1}{\bar{\eta}_1}$$

$$\frac{p_2 - c_2 - c_0}{p_2} = \frac{\lambda}{1 + \lambda} \frac{1}{\bar{\eta}_2}$$

where  $\bar{\eta}_1, \bar{\eta}_2$  are superelasticity formulas for the hybrid population of consumers who bypass and those who do not.

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<sup>11</sup>Note that if in addition a direct subsidy (or tax) for bypass was available this effect would disappear.

The deviation of the access price from the marginal cost is then :

$$\frac{a - c_0}{p_2} = \frac{\lambda}{1 + \lambda} \left\{ \frac{1}{\bar{\eta}_2} - \frac{1}{\eta_2(a)} - \frac{\eta_{21}(a)}{\bar{\eta}_1 \eta_2(a)} \right\}.$$

If elasticities are constant, if the proportion of those who bypass is small (then  $\bar{\eta}_2 \approx \hat{\eta}_2$  where  $\hat{\eta}_2$  is the classical super-elasticity and similarly  $\bar{\eta}_1 \approx \hat{\eta}_1$ )<sup>12</sup>, we obtain the approximation :

$$a \simeq c_0.$$

Pricing access at marginal cost is then appropriate.

We have assumed here the existence of transfers. If there is no transfer, similar results hold as long as the tax revenue on good 2 goes to the monopoly. If not, access prices participate strongly in raising funds to pay for the fixed costs of the local loop and very inefficient bypass is to be expected.

## 9 Access Pricing and Entry : The Efficient Component Pricing Rule.

So far we have focussed on pricing access when there exist competitors for some services which need access. The concern was the efficiency of supply of services given existing firms. In other words we have assumed that the regulated firm's rival enters regardless of the access price. Suppose now that the competitor must pay a fixed entry or operating cost. The optimal access pricing rule may be affected by the existence of this fixed cost, because the entrant does not internalize the consumer surplus created by the introduction of good 2. Ideally one would like to use direct entry subsidies to deal optimally with the entry decision. [Under incomplete information about the entrant's cost, one would need a nonlinear transfer function of the quantity to be produced if the asymmetry of information is about the variable cost, and a stochastic decision of entry as a function of the announced fixed cost if the asymmetry of information is about the fixed cost]. In the absence of such alternative instruments it may be optimal to lower the access price obtained so far to induce a better entry decision<sup>13</sup>.

Also, concerned with entry decisions, Baumol has proposed a simple and influential

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<sup>12</sup>In the absence of these assumptions different taxes for those who bypass and those who do not would be desirable.

<sup>13</sup>The relatively favorable interconnect prices levied on Mercury to access British Telecom's network are a dynamic version of this point. It is widely believed that they were designed to realise a particular market structure. A different but related reason for low access prices is the high switching costs of consumers.



access pricing rule <sup>14</sup>, called the “efficient component pricing rule” (ECP rule) which follows from the precepts of contestability theory.

In our notation, this rule recommends an access price equal to the difference between the monopoly’s price and its marginal cost in the competitive market :

$$a = p_1 - c_1. \quad (46)$$

The idea is that an entrant with marginal cost  $c$  on the competitive segment will enter if and only if it is more efficient :

$$p_2 = a + c = (p_1 - c_1) + c < p_1 \Leftrightarrow c < c_1.$$

• **ECP rule under perfect substitutes** : The following assumptions make this rule optimal. First, the monopoly’s and the entrant’s goods are perfect substitutes. Otherwise entry by a less efficient entrant may be desirable. Second, the regulator observes the monopoly’s marginal cost on the competitive market. Third, the entrant has no monopoly power. Otherwise, the choice of  $p_1$  not only determines the access price, but also constrains the entrant’s monopoly power. Fourth, and relatedly, the technologies exhibit constant returns to scale. Fixed entry costs for instance would create several difficulties with the Baumol rule. On the one hand, the no-monopoly-power assumption would be stretched. On the other hand, the entrant might not enter because he does not internalise the increase in consumer surplus created by entry. Fifth, the benchmark pricing rule is marginal cost pricing. [Incidentally, Baumol seems to recommend Ramsey pricing as the benchmark. But,  $p_2 = a + c$  is lower than the Ramsey price for cost  $(c_0 + c)$  if  $p_1$  is the Ramsey price for cost  $(c_0 + c_1)$ .] On the other hand, if those five conditions are satisfied, it can be argued that the access pricing rule (46) is irrelevant. Either  $c < c_1$  and the monopoly should supply only access ( $p_1$  is irrelevant) ; or  $c > c_1$  and no access should be given ( $a$  is irrelevant).

• **ECP rule under imperfect substitutes** : In the rest of this section we introduce differentiation (as we have done until now) to justify the presence of several firms in the competitive segment. To retain the spirit of the Baumol rule as much as possible, we assume constant returns to scale, competitive entrants, known marginal cost for the entrants, and the dichotomy property for the monopoly’s cost function. For instance, we can take the sub-cost functions assumed in Section 5. The dichotomy ensures that the pricing rules will not be affected by incentive corrections.

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<sup>14</sup>See Baumol (1993) : “If a component of a product is offered by a single supplier who also competes with others in offering the remaining product component, the single-supplier component’s price should cover its incremental cost plus the opportunity cost incurred when a rival supplies the final product”.

In our model the Baumol rule amounts to :

$$a(\beta) = p_1(\beta) - c_1 = p_1(\beta) - \frac{C_1}{q_1}.$$

Competition in the competitive segment implies that  $p_2(\beta) = p_1(\beta) + c - c_1$ .

Optimizing expected social welfare under constraint (46) and under the incentive constraints (see appendix 6), we obtain the access pricing rule

$$a = C_{0Q} + \frac{\bar{\lambda}(\beta)}{1 + \bar{\lambda}(\beta) \hat{\eta}_2^*} \frac{p_2}{\hat{\eta}_2^*}, \quad (47)$$

where  $\hat{\eta}_2^*$  is an "average elasticity"

$$\hat{\eta}_2^* = \frac{p_2}{p_1} \frac{q_1}{q_1 + q_2} \eta_1 + \frac{q_2}{q_1 + q_2} \eta_2 - \frac{p_2}{p_1} \frac{q_2}{q_1 + q_2} \eta_{21} - \frac{q_1}{q_1 + q_2} \eta_{12}. \quad (48)$$

Compared to the formula  $a = C_{0Q} + \frac{\bar{\lambda}(\beta)}{1 + \bar{\lambda}(\beta) \hat{\eta}_2} \frac{p_2}{\hat{\eta}_2}$  obtained in Section 5 (in the case assumed here of dichotomy), the elasticity used is not the correct one, namely

$$\hat{\eta}_2 = \eta_2 \frac{\eta_1 \eta_2 - \eta_{12} \eta_{21}}{\eta_1 \eta_2 + \eta_2 \eta_{21}}.$$

The difference between the two formulas (assuming that there is no need for an incentive correction) stems from the fact that the Baumol rule ties the two prices on the competitive segment. Let us compare the two rules in some specific cases :

- *Symmetric demands and costs.* One then has  $\hat{\eta}_1 = \hat{\eta}_2$  and  $c = c_1$ . These imply that  $p_2 - c - c_0 = p_1 - c_1 - c_0$ , or

$$a = p_1 - c_1.$$

Thus for symmetric demands and costs, the efficient component pricing rule is consistent with optimal regulation.

- *Linear demands, symmetric costs, and captive customers.* Let us alter the previous symmetry assumption in one respect : the monopoly has captive customers while competitors do not. Under linear demands, the demand functions are

$$q_1 = a_1 - bp_1 + dp_2$$

$$q_2 = a_2 - bp_2 + dp_1$$

with  $d < b$  and  $a_2 < a_1$ . Marginal costs are identical :  $c_1 = c$ . Our formulas then yield

$$p_1 > p_2 \quad \text{and} \quad a < p_1 - c_1.$$



The intuition is that the monopoly must charge a higher price than its competitors because captive customers make markup relatively efficient in covering the fixed cost. In turn, this high markup on good 1 can be viewed as an access subsidy relative to the efficient component pricing rule.

- *Linear symmetric demands, cost superiority of monopoly.* Let us assume in the previous example that  $a_1 = a_2$ , but  $c_1 < c$ . Then our formulas yield :

$$p_1 < p_2 \quad \text{and} \quad a < p_1 - c_1.$$

The intuition is now that under linear demand the price differential  $p_2 - p_1$  must only partially reflect the cost differential  $c_2 - c_1$ . There is "cost absorption". The access price must therefore be lower than that recommended the efficient component pricing rule.

**Remark on Price Caps :** Suppose that the monopoly is regulated according to the price cap :

$$\alpha_0 p_0 + \alpha_1 p_1 \leq \bar{p}, \quad (49)$$

with the access pricing rule :

$$a = p_1 - c_1$$

or

$$p_2 = p_1 - c_1 + c.$$

This of course is not pure price cap regulation (neither is practice) because the regulator makes use of the cost information  $c_1$ .

If the weights  $\alpha_0$  and  $\alpha_1$  are chosen approximately equal to  $q_0$  and  $q_1 + q_2$  respectively we obtain (see appendix 7)

$$\frac{p_0 - C_{0Q}}{p_0} = \frac{(1 - \nu)}{\eta_0}$$

$$\frac{p_1 - C_{0Q} - C_{1q_1}}{p_1} = \frac{1 - \nu}{\hat{\eta}_2^*},$$

where  $\nu$  is the multiplier of the price cap constraint and  $\hat{\eta}_2^*$  is given by (49). Then

$$a = C_{0Q} + \frac{(1 - \nu)p_1}{\hat{\eta}_2^*}.$$

Comparing with (48) we notice two differences. First  $1 - \nu$  will in general differ from  $\frac{\hat{\lambda}(\beta)}{1 + \lambda(\beta)}$ . On the other hand, under a price cap, effort is optimal conditionally on the total production level  $q_0 + q_1 + q_2$  and therefore  $C_{0Q}$  is evaluated at a (conditionally) higher effort level than under optimal regulation.

## 10 Access Pricing in Practice

To assess current practice in the light of the theoretical model, assume that

$$C_0 = c_0Q + k_0, C_1 = c_1q_1 \text{ and } C_2 = cq_2.$$

a) **Fully distributed costs** : The most popular method of pricing access consists in allocating the fixed cost to the firm's products in a mechanistic way. For instance, a product's markup above marginal cost may be uniform :

$$p_0 = c_0 + \frac{k_0}{Q}, p_1 = (c_1 + c_0) + \frac{k_0}{Q}, a = c_0 + \frac{k_0}{Q}.$$

This accounting rule generates "excess" revenue  $\frac{k_0q_0}{Q} + \frac{k_0q_1}{Q} + \frac{k_0q_2}{Q} = k_0$ , that covers the fixed cost.

It is interesting to note that this accounting rule satisfies the efficient component pricing rule, as

$$p_1 - c_1 = a.$$

This of course need not be the case for alternative methods of distributing the fixed cost. For example, a markup proportional to marginal cost, namely

$$p_0 = c_0(1 + \delta), p_1 = (c_0 + c_1)(1 + \delta), a = c_0(1 + \delta),$$

where  $\delta$  is chosen so as to ensure budget balance, yields

$$p_1 - c_1 > a.$$

There is no point dwelling on the conceptual drawbacks of fully distributed costs. Let us just recall that the fixed cost allocation is arbitrary and has no reason to reflect the proper cost, demand and entry considerations.

b) **The Oftel rule** : The access rule designed by Oftel for the access to British Telecom's (BT) network can be sketched as follows : Let  $B_0 = (p_0 - c_0)q_0$ ,  $B_1 = (p_1 - c_0 - c_1)q_1$ , and  $B_2 = (a - c_0)q_2$  denote BT's profits in its various product lines (local, long distance, access). The "access deficit" AD is here equal to the fixed cost  $k_0$ .

The idea of the rule is to set a *usage-based* price of access to BT's local network. Namely the margin above the cost of giving access is proportional to the access deficit per unit of BT's output in this activity and to the share of BT's variable profits provided by this activity. For the long distance activity, one thus has :

$$a - c_0 = \frac{AD}{q_1} \frac{B_1}{B_0 + B_1 + B_2} \quad (50)$$

Note that  $B_2$  itself depends on the access price and that (as is the case for the determination of access prices under fully distributed costs) formula (50) involves a fixed point. In practice (again as in the case of fully distributed costs), the access price must be the outcome of a dynamic tâtonnement, whose path ought to be studied in more detail.

If the budget is balanced (that is, if  $AD = B_0 + B_1 + B_2$ ), the Oftel formula interestingly reduces to

$$a = p_1 - c_1$$

which is exactly the “opportunity cost” of BT in the activity. The Oftel formula thus yields under budget balance the efficient component pricing rule.

To see that the access pricing rule is usage -and not only cost- based, suppose that there is competition not only on the long distance domestic market (goods  $q_1$  and  $q_2$ ), but also on the international market (good  $q_3$  produced by BT and good  $q_4$  produced by its competitors). The Oftel rule then defines different access prices for competitors,  $a_2$  on the domestic long distance market and  $a_4$  on the international market despite identical access costs<sup>15</sup>. With obvious notation,

$$a_2 - c_0 = \frac{AD}{q_1} \frac{B_1}{\sum_{i=0}^4 B_i} \quad \text{and} \quad a_4 - c_0 = \frac{AD}{q_3} \frac{B_3}{\sum_{i=0}^4 B_i},$$

and so

$$\frac{a_2 - c_0}{a_4 - c_0} = \frac{B_1/q_1}{B_3/q_3}. \quad (51)$$

The markups are thus proportional to BT’s unit revenues on the competitive segments. The Oftel rule is therefore related to our access pricing formula, in that both are usage-based and reflect the loss of revenue imposed by competitors on the network provider. The difference between the Oftel rule and our formula (29) is that the unit revenues replace the superelasticities.

## 11 Conclusion

In a first best world, access pricing to a network (like any other pricing) should be marginal cost pricing. In practice the appropriate rule departs from the first best and depends on the constraints and on the available instruments.

First, the provision of the network imposes fixed costs which cannot be financed by nondistortive lump sum taxes. Competitors should then contribute to the fixed cost of

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<sup>15</sup>For the sake of the argument. In practice, a domestic call uses two domestic local loops while an international call uses only one. Further, the access price peak load structure can only be coarse, and therefore network costs may differ for the two types of calls if their timing structures differ within pricing bands.



the network. This contribution, which takes the form of an access price above marginal cost, reduces either the financial burden of the regulator (in the case of transfers) or the price distortions associated with the firm's budget constraint.

Second, asymmetric information of the regulator about the firm's costs may introduce other standard distortions, because informational rents must be taken into account when defining proper marginal costs. Further distortions may be desirable if network and competitive segment subcosts cannot be disentangled.

When taxation of the competitive goods is feasible, access can be priced at marginal cost while taxes contribute to covering the fixed cost. [Note nevertheless that the marginal cost referred to is the marginal cost which arises from the optimal regulation of the firm, and is in general different from the first best marginal cost because effort is lower]. This disconnection of consumer and producer prices is very handy if consumers on the competitive segment can bypass the network. By contrast, a separation of regulatory and taxation mandates reduces the regulators' number of instruments, and the access price must then arbitrate very imperfectly between the goals of limiting inefficient bypass and participating to the coverage of fixed costs. It is clear more generally that access pricing rules become more and more complex as well as inefficient as the regulator tries to use access prices, rather than complementary instruments, in order to meet various market structure goals such as inducing proper entry or reducing monopoly power in the competitive segment or to achieve distributive objectives.

Last, we have shown that our analysis can shed substantial light on the effectiveness of policy proposals such as the efficient component pricing rule and of existing policies such as the Oftel rule for British Telecom.

Looking forward, our analysis can be pursued in several directions. With a proper regulatory model of predatory behavior, one should be able to investigate the conjecture that access pricing is a more efficient tool to prey than the pricing of the competitive products. The intuition is the following : A low price for the competitive product signals a high efficiency. This information can be used by the regulator to extract the monopoly's rent in the future. On the other hand, a high access price seems immune to ratcheting.

Next, one could substitute to the optimal regulation various types of current regulations (price cap, rate of return) and analyze how the deviations from optimal regulation impact on the access pricing formulas (see for example the Remark in Section 9). One could even depart from the regulatory context and see how competition policy alone would deal with access pricing. Excessive access prices could be considered as a predatory tool to eliminate competition as has been claimed in the AT&T case before divestiture.

Our framework also enables us to discuss a current debate in Europe. Telecommunication companies are required to offer universal service of local telephone (here good zero)

at a low price. Their budget constraint would normally force them to increase the price of their other services as well as the price of access to the network. A concurrent low-access-charge constraint (for example the European Community might demand marginal cost pricing in order to increase competition) then forces the network firm to further increase the price of its other services and to lose its competitive edge. We are not then very far from the solution adopted by the U.S. courts, which prohibits the network from offering competitive services. There, universal service is financed by appropriate access charges to the network instead of the profits made by the network firm on its additional services as in Europe.

We have not discussed nonlinear access pricing rules, for example two-part tariffs. As long as the fixed part of the tariff plays the role of a lump sum tax, it is an excellent tool to raise the money needed to pay for the fixed cost of the network. However, one must take in account the fact that beyond some level the fixed charge lowers the connection rate (see for example Laffont-Tirole (1993) ch. 2). Similarly we have not discussed the important practical issue of peak load access pricing.

Finally, let us stress that, to a large extent, we have left aside various dynamic issues : investments (in particular unobservable specific ones) for network development (see however Remark 4 in Section 3), consequences of regulators' limited commitment power, as well as regulation of entry. Inefficient developement of the network, more costly rents of asymmetric information, and inadequate entry behavior are then to be expected. These questions can be addressed within our general regulatory framework (see ch. 1, 9, and 13 in Laffont-Tirole (1993) for the basic principles).

## APPENDIX 1 : Optimal regulatory rule (Section 3)

Let  $\mu(\beta)$  be the co-state variable associated with  $U$ . From the Pontryagin principle

$$\dot{\mu}(\beta) = \lambda f(\beta).$$

Using the transversality condition  $\mu(\underline{\beta}) = 0$  we have

$$\mu(\beta) = \lambda F(\beta).$$

The differentiation of the Hamiltonian

$$\begin{aligned} H = & \left\{ S(q_0(p_0(\beta))) + V(q_1(p_1(\beta), p_2(\beta)), q_2(p_1(\beta), p_2(\beta))) \right. \\ & \left. + \lambda [p_0(\beta)q_0(p_0(\beta)) + p_1(\beta)q_1(p_1(\beta), p_2(\beta)) + p_2(\beta)q_2(p_1(\beta), p_2(\beta))] \right. \\ & - (1 + \lambda) [\psi(e_0(\beta) + e_1(\beta)) + C_0(\beta, e_0(\beta), q_0(p_0(\beta)) + q_1(p_1(\beta), p_2(\beta)) + q_2(p_1(\beta), p_2(\beta))) \\ & \left. + C_1(\beta, e_1(\beta), q_1(p_1(\beta), p_2(\beta))) + c q_2(p_1(\beta), p_2(\beta))] - \lambda U(\beta) \right\} f(\beta) \\ & - \mu(\beta) \psi'(e_0(\beta) + e_1(\beta)) \left\{ \frac{\partial E_0}{\partial \beta} (\beta, C_0(\beta, e_0(\beta), q_0(p_0(\beta)) + q_1(p_1(\beta), p_2(\beta)) + q_2(p_1(\beta), p_2(\beta))), \right. \\ & \left. q_0(p_0(\beta)) + q_1(p_1(\beta), p_2(\beta)) + q_2(p_1(\beta), p_2(\beta))) \right. \\ & \left. + \frac{\partial E_1}{\partial \beta} (\beta, C_1(\beta, e_1(\beta), q_1(p_1(\beta), p_2(\beta))), q_1(p_1(\beta), p_2(\beta))) \right\} \end{aligned}$$

with respect to  $p_0$ ,  $p_1$ ,  $p_2$  gives (24), (25) and (26) and with respect to  $e_0$ ,  $e_1$  gives (30) and (31).

## APPENDIX 2 : Sub-cost observability

Part (i) results from (37). To prove that sub-cost observation is useless when  $\partial^2 E_k / \partial C_k \partial \beta = 0$  for  $k = 1, 2$  note that, at the optimal regulatory policy,  $(-\partial E_k / \partial C_k)$  is the same for all  $k$ . That is, given the firm's scheme under sub-cost observability,  $\beta \rightarrow \{t(\beta), C_0(\beta), C_1(\beta), Q(\beta), q_1(\beta)\}$  consider the associated incentive scheme  $\beta \rightarrow \{t(\beta), C(\beta), Q(\beta), q_1(\beta)\}$  under sub-cost unobservability, where  $C(\beta) = C_0(\beta) + C_1(\beta)$ . This incentive scheme is incentive compatible. To show this, it suffices to show that if type  $\beta$  announces  $\tilde{\beta}$  it will choose the sub-cost allocation  $\{C_0(\beta), C_1(\beta)\}$  corresponding to its announcement, even though the regulator cannot control sub-costs. Note that from (37), there exists  $\alpha$  such that

$$\frac{\partial E_k}{\partial C_k}(\tilde{\beta}, C_k(\tilde{\beta}), Q_k(\tilde{\beta})) = \alpha \quad k = 0, 1$$

and

$$C_0(\tilde{\beta}) + C_1(\tilde{\beta}) = C(\tilde{\beta}),$$

where  $Q_0 \equiv Q$  and  $Q_1 = q_1$ .

But the condition  $\partial^2 E_k / \partial C_k \partial \beta = 0$  implies that  $\frac{\partial E_k}{\partial C_k}(\tilde{\beta}, C_k(\tilde{\beta}), Q_k(\tilde{\beta})) = \alpha$  for all  $k$ , as well.

From our assumption,  $\{C_0(\tilde{\beta}) + C_1(\tilde{\beta}) = C(\tilde{\beta})\}$  is the unique cost minimizer for type  $\beta$  when it announces  $\tilde{\beta}$ . We can thus implement the same allocation when sub-costs are unobservable as when they are.



### APPENDIX 3 : Competitor market power

Because of the dichotomy assumption, the optimization with respect to the access price can be carried out under complete information :

$$\max_{p_0, p_1, p_2} \left\{ S(q_0(p_0)) + V(q_1(p_1, p_2), q_2(p_1, p_2)) - p_0 q_0(p_0) - p_1 q_1(p_1, p_2) - (a + c) q_2(p_1, p_2) \right\} \quad (A3.1)$$

s.t.

$$\begin{aligned} p_0 q_0(p_0) + p_1 q_1(p_1, p_2) + a q_2(p_1, p_2) - c_0(q_0(p_0) + q_1(p_1, p_2) + q_2(p_1, p_2)) \\ - c_1 q_1(p_1, p_2) \geq \psi(e_0 + e_1) \end{aligned} \quad (A3.2)$$

$$q_2(p_1, p_2) + p_2 \frac{\partial q_2}{\partial p_2}(p_1, p_2) - (a + c) \frac{\partial q_2}{\partial p_2}(p_1, p_2) = 0. \quad (A3.3)$$

From (A3.3)

$$a + c = p_2 + \frac{q_2(p_1, p_2)}{\frac{\partial q_2}{\partial p_2}(p_1, p_2)} = p_2(1 - 1/\eta_2(p_1, p_2)).$$

Substituting into (A3.1), (A3.2) we have

$$\max_{p_0, p_1, p_2} \left\{ S(q_0(p_0)) + V(q_1(p_1, p_2), q_2(p_1, p_2)) - p_0 q_0(p_0) - p_1 q_1(p_1, p_2) - p_2 q_2(p_1, p_2) + \frac{p_2 q_2(p_1, p_2)}{\eta_2(p_1, p_2)} \right\}$$

s.t.

$$\begin{aligned} p_0 q_0(p_0) + p_1 q_1(p_1, p_2) + p_2 q_2(p_1, p_2) - c_0(q_0(p_0) + q_1(p_1, p_2) + q_2(p_1, p_2)) \\ - c_1 q_1(p_1, p_2) - c_2 q_2(p_1, p_2) \geq \frac{p_2 q_2(p_1, p_2)}{\eta_2(p_1, p_2)} + \psi(e_0 + e_1). \end{aligned}$$

Let  $1 + \tilde{\lambda}(\beta)$  denote the multiplier of the constraint. We substitute  $\tilde{\lambda}$  to  $\tilde{\lambda}(\beta)$  below for convenience.

Maximizing with respect to  $(p_1, p_2)$  we obtain :

$$\begin{aligned} \frac{\partial V}{\partial q_1} \frac{\partial q_1}{\partial p_1} + \frac{\partial V}{\partial q_2} \frac{\partial q_2}{\partial p_1} - q_1 - p_1 \frac{\partial q_1}{\partial p_1} - p_2 \frac{\partial q_2}{\partial p_1} \\ + (1 + \tilde{\lambda}) \left[ p_1 \frac{\partial q_1}{\partial p_1} + q_1 + p_2 \frac{\partial q_2}{\partial p_1} - c_0 \left( \frac{\partial q_1}{\partial p_1} + \frac{\partial q_2}{\partial p_1} \right) \right] \end{aligned}$$



$$-c_1 \frac{\partial q_1}{\partial p_1} - c \frac{\partial q_2}{\partial p_1} \Big] - \bar{\lambda} \frac{\partial}{\partial p_1} \left( \frac{p_2 q_2}{\eta_2} \right) = 0$$

$$\begin{aligned} & \frac{\partial V}{\partial q_1} \frac{\partial q_1}{\partial p_2} + \frac{\partial V}{\partial q_2} \frac{\partial q_2}{\partial p_2} - \frac{\partial q_1}{\partial p_2} - q_2 - p_2 \frac{\partial q_2}{\partial p_2} \\ & + (1 + \bar{\lambda}) \left[ p_1 \frac{\partial q_1}{\partial p_2} + q_2 + p_2 \frac{\partial q_2}{\partial p_2} - c_0 \left( \frac{\partial q_1}{\partial p_2} + \frac{\partial q_2}{\partial p_2} \right) \right. \end{aligned}$$

$$\left. -c_1 \frac{\partial q_1}{\partial p_2} - c \frac{\partial q_2}{\partial p_2} \right] - \bar{\lambda} \frac{\partial}{\partial p_2} \left( \frac{p_2 q_2}{\eta_2} \right) = 0$$

$$\begin{bmatrix} \frac{\partial q_1}{\partial p_1} & \frac{\partial q_2}{\partial p_1} \\ \frac{\partial q_1}{\partial p_2} & \frac{\partial q_2}{\partial p_2} \end{bmatrix} \begin{bmatrix} p_1 - c_0 - c_1 \\ p_2 - c_0 - c \end{bmatrix} = \frac{\bar{\lambda}}{1 + \bar{\lambda}} \begin{bmatrix} -q_1 + \frac{\partial}{\partial p_1} \left( \frac{p_2 q_2}{\eta_2} \right) \\ -q_2 + \frac{\partial}{\partial p_2} \left( \frac{p_2 q_2}{\eta_2} \right) \end{bmatrix}$$

$$\begin{bmatrix} p_1 - c_0 - c_1 \\ p_2 - c_0 - c \end{bmatrix} = \frac{\bar{\lambda}}{1 + \bar{\lambda}} \begin{bmatrix} \frac{p_1}{\hat{\eta}_1} \\ \frac{p_2}{\hat{\eta}_2} \end{bmatrix} + \frac{\bar{\lambda}}{1 + \bar{\lambda}} \begin{bmatrix} \frac{\partial q_1}{\partial p_1} & \frac{\partial q_2}{\partial p_1} \\ \frac{\partial q_1}{\partial p_2} & \frac{\partial q_2}{\partial p_2} \end{bmatrix}^{-1} \begin{bmatrix} \frac{\partial}{\partial p_1} \left( \frac{p_2 q_2}{\eta_2} \right) \\ \frac{\partial}{\partial p_2} \left( \frac{p_2 q_2}{\eta_2} \right) \end{bmatrix}.$$

The second term of the right-hand side equals

$$\begin{aligned} & \frac{\bar{\lambda}}{1 + \bar{\lambda}} \frac{1}{\frac{\partial q_1}{\partial p_1} \cdot \frac{\partial q_2}{\partial p_2} - \frac{\partial q_1}{\partial p_2} \cdot \frac{\partial q_2}{\partial p_1}} \\ & \begin{bmatrix} \frac{p_2}{\eta_2} \left( \frac{\partial q_2}{\partial p_2} \cdot \frac{\partial q_2}{\partial p_1} - \frac{\partial q_2}{\partial p_1} \cdot \frac{\partial q_2}{\partial p_2} \right) + p_2 q_2 \left[ \frac{\partial q_2}{\partial p_2} \frac{\partial}{\partial p_1} \left( \frac{1}{\eta_2} \right) - \frac{\partial q_2}{\partial p_1} \frac{\partial}{\partial p_2} \left( \frac{1}{\eta_2} \right) \right] - \frac{\partial q_2}{\partial p_1} \frac{q_2}{\eta_2} \\ \frac{p_2}{\eta_2} \left( \frac{\partial q_1}{\partial p_1} \cdot \frac{\partial q_2}{\partial p_2} - \frac{\partial q_1}{\partial p_2} \cdot \frac{\partial q_2}{\partial p_1} \right) + p_2 q_2 \left[ \frac{\partial q_1}{\partial p_1} \frac{\partial}{\partial p_2} \left( \frac{1}{\eta_2} \right) - \frac{\partial q_1}{\partial p_2} \frac{\partial}{\partial p_1} \left( \frac{1}{\eta_2} \right) \right] + \frac{\partial q_1}{\partial p_1} \frac{q_2}{\eta_2} \end{bmatrix} \\ & = \frac{\bar{\lambda}}{1 + \bar{\lambda}} \begin{bmatrix} -\frac{\frac{R_2}{q_1} \left[ p_1 \eta_2 \frac{\partial}{\partial p_1} \left( \frac{1}{\eta_2} \right) + p_2 \eta_{21} \frac{\partial}{\partial p_2} \left( \frac{1}{\eta_2} \right) \right] + \frac{R_2 \eta_{21}}{q_1 \eta_2}}{\eta_1 \eta_2 - \eta_{12} \eta_{21}} \\ \frac{p_2}{\eta_2} - \frac{\frac{R_2}{q_2} \left[ p_2 \eta_1 \frac{\partial}{\partial p_2} \left( \frac{1}{\eta_1} \right) + p_1 \eta_{12} \frac{\partial}{\partial p_1} \left( \frac{1}{\eta_2} \right) \right] + \frac{R_2 \eta_{11}}{q_2 \eta_2}}{\eta_1 \eta_2 - \eta_{12} \eta_{21}} \end{bmatrix} \end{aligned}$$

$$\frac{p_1 - c_0 - c_1}{p_1} = \frac{\tilde{\lambda}}{1 + \tilde{\lambda}} \left[ \frac{1}{\hat{\eta}_1} - \frac{\frac{R_2}{R_1} \left[ p_1 \eta_2 \frac{\partial}{\partial p_1} \left( \frac{1}{\eta_2} \right) + p_2 \eta_{21} \frac{\partial}{\partial p_2} \left( \frac{1}{\eta_2} \right) \right] + \frac{\eta_{12}}{\eta_2}}{\eta_1 \eta_2 - \eta_{12} \eta_{21}} \right]$$

$$\frac{p_2 - c_0 - c}{p_2} = \frac{\tilde{\lambda}}{1 + \tilde{\lambda}} \left[ \frac{1}{\hat{\eta}_2} + \frac{1}{\eta_2} - \frac{\left[ p_2 \eta_1 \frac{\partial}{\partial p_2} \left( \frac{1}{\eta_2} \right) + p_2 \eta_{12} \frac{\partial}{\partial p_1} \left( \frac{1}{\eta_2} \right) \right] + \frac{\eta_1}{\eta_2}}{\eta_1 \eta_2 - \eta_{12} \eta_{21}} \right],$$

where  $R_k \equiv p_k q_k$ .

Since  $a = p_2 - c - \frac{p_2}{\eta_2}$ , we have

$$a = C_{0Q} - \frac{p_2}{\eta_2} + \frac{\tilde{\lambda}(\beta)}{1 + \tilde{\lambda}(\beta)} p_2 \frac{1}{\eta_2^*} - \frac{\tilde{\lambda}(\beta)}{1 + \tilde{\lambda}(\beta)} \frac{p_2 \eta_1 \frac{\partial}{\partial p_2} \left( \frac{1}{\eta_2} \right) + p_1 \eta_{12} \frac{\partial}{\partial p_1} \left( \frac{1}{\eta_2} \right)}{\eta_1 \eta_2 - \eta_{12} \eta_{21}} p_2$$

with

$$\eta_2^* = \eta_2 \cdot \frac{\eta_1 \eta_2 - \eta_{12} \eta_{21}}{2\eta_1 \eta_2 + \eta_{21} \eta_2 - \eta_{21} \eta_{12} - \eta_1}.$$

## APPENDIX 4 : Restricted regulation

The dichotomy implies that price formulae can be obtained as in the full information case. Let

$$W \equiv S(q_0) + V(q_1, q_2) - p_0 q_0 - p_1 q_1 - p_2 q_2$$

denote the consumer net surplus, and

$$MP \equiv q_1 + \frac{\partial q_1}{\partial p_1} [p_1 - c_0 - c_1] + \frac{\partial q_2}{\partial p_1} [p_2 - c_0 - c]$$

denote the monopolist's marginal profit and let  $P$  denote its profit. The planner solves :

$$\max_{\{p_0, p_1, p_2\}} W$$

subject to

$$(BB) \quad P - \psi = p_0 q_0 + p_1 q_1 + (p_2 - c) q_2 - c_0 q_0 - c_1 q_1 - \psi = 0 \text{ and}$$

$$MP = 0.$$

Let  $(1 + \tilde{\lambda})$  and  $\nu$  denote the shadow prices of the two constraints. Because the left-hand side of  $(BB)$  is maximized with respect to  $p_1$ , the first-order condition with respect to  $p_1$  is :

$$\frac{dW}{dp_1} + \nu \left( \frac{dMP}{dp_1} \right) = 0.$$

Because the net consumer surplus decreases with prices, and from our concavity assumption on profit, we obtain  $\nu < 0$ . The derivative of the Lagrangian with respect to  $p_2$  yields

$$\frac{dW}{dp_2} + (1 + \tilde{\lambda}) \frac{dP}{dp_2} + \nu \frac{dMP}{dp_2} = 0.$$

The first-two terms are the standard terms obtained in the absence of monopoly pricing. The correction in the access pricing formula ( $a = p_2 - c$ ) depends on the sign of  $d(MP)/dp_2$ , which is positive under generalized strategic complementarity. This last equation shows that for a given  $p_1$  the price  $p_2$ , and therefore the access price, should be lowered. But  $p_1$  is also increased and the result ambiguous. However, for linear demand functions and constant returns to scale both effects cancel, giving the same access pricing formula as in the case of full regulation.

## APPENDIX 5

Let  $\theta^*$  be defined by  $\theta^* = \hat{V}(p_1, p_2 + b - a) - \hat{V}(p_1, p_2)$ .

Hence

$$\begin{aligned}\frac{\partial \theta^*}{\partial p_1} &= q_1 - q_1(a) \\ \frac{\partial \theta^*}{\partial p_2} &= q_2 - q_2(a) \\ \frac{\partial \theta^*}{\partial a} &= q_2(a).\end{aligned}$$

Let  $\Delta R = (p_1 - c_1 - c_0)(q_1(a) - q_1) + (p_2 - a - c_2)q_2(a) - (p_2 - c_2 - c_0)q_2$ .

Maximizing (45) with respect to  $p_1, p_2, a$  yields :

$$\begin{aligned}G(\theta^*) &\left[ \lambda q_1(a) + (1 + \lambda) \left[ (p_1 - c_1 - c_0) \frac{\partial q_1}{\partial p_1}(a) + (p_2 - a - c_2) \frac{\partial q_2}{\partial p_1}(a) \right] \right] \\ &+ (1 - G(\theta^*)) \left[ \lambda q_1 + (1 + \lambda) \left[ (p_1 - c_1 - c_0) \frac{\partial q_1}{\partial p_1} + (p_2 - c_0 - c_2) \frac{\partial q_2}{\partial p_1} \right] \right] \\ &+ g(\theta^*)(q_1 - q_1(a))(1 + \lambda) \Delta R = 0\end{aligned}\tag{A5.1}$$

$$\begin{aligned}G(\theta^*) &\left[ \lambda q_2(a) + (1 + \lambda) \left[ (p_1 - c_1 - c_0) \frac{\partial q_1}{\partial p_2}(a) + (p_2 - a - c_2) \frac{\partial q_2}{\partial p_2}(a) \right] \right] \\ &+ (1 - G(\theta^*)) \left[ \lambda q_2 + (1 + \lambda) \left[ (p_1 - c_1 - c_0) \frac{\partial q_1}{\partial p_2} + (p_2 - c_2 - c_0) \frac{\partial q_2}{\partial p_2} \right] \right] \\ &+ g(\theta^*)[q_2 - q_2(a)](1 + \lambda) \Delta R = 0\end{aligned}\tag{A5.2}$$

$$\begin{aligned}G(\theta^*) &\left[ -\lambda q_2(a) - (1 + \lambda) \left[ (p_1 - c_1 - c_0) \frac{\partial q_1}{\partial p_2}(a) + (p_2 - a - c_2) \frac{\partial q_2}{\partial p_2}(a) \right] \right] \\ &+ g(\theta^*)q_2(a)(1 + \lambda) \Delta R = 0\end{aligned}\tag{A5.3}$$

Hence

$$\begin{aligned}&G(\theta^*) \left[ \lambda q_1(a) + (1 + \lambda)(p_1 - c_1 - c_0) \frac{\partial q_1}{\partial p_1}(a) \right. \\ &\quad \left. - \left( \lambda q_2(a) + (1 + \lambda)(p_1 - c_1 - c_0) \frac{\partial q_1}{\partial p_2}(a) \right) \frac{\partial q_2 / \partial p_1(a)}{\partial q_2 / \partial p_2(a)} \right] \\ &+ (1 - G(\theta^*)) \left[ \lambda q_1 + (1 + \lambda) \left[ (p_1 - c_1 - c_0) \frac{\partial q_1}{\partial p_1} + (p_2 - c_0 - c_2) \frac{\partial q_2}{\partial p_1} \right] \right] \\ &+ g(\theta^*)(1 + \lambda) \Delta R \left[ q_1 - q_1(a) - q_2(a) \frac{\partial q_2 / \partial p_1(a)}{\partial q_2 / \partial p_2(a)} \right] = 0,\end{aligned}$$



and

$$[1 - G(\theta^*)] \left[ \lambda q_2 + (1 + \lambda) \left[ (p_1 - c_1 - c_0) \frac{\partial q_1}{\partial p_2} + (p_2 - c_2 - c_0) \frac{\partial q_2}{\partial p_2} \right] \right] + g(\theta^*) q_2 (1 + \lambda) \Delta R = 0.$$

Given the ambiguity of the sign of  $\Delta R$ , we will consider the case  $\Delta R = 0$ .

$$(1 + \lambda) \begin{bmatrix} G \left( \frac{\partial q_1}{\partial p_1}(a) - \frac{\partial q_1}{\partial p_2}(a) \frac{\frac{\partial q_2}{\partial p_1}(a)}{\frac{\partial q_2}{\partial p_2}(a)} \right) + (1 - G) \frac{\partial q_1}{\partial p_1} & (1 - G) \frac{\partial q_2}{\partial p_1} \\ (1 - G) \frac{\partial q_1}{\partial p_2} & (1 - G) \frac{\partial q_2}{\partial p_2} \end{bmatrix} \begin{bmatrix} (p_1 - c_1 - c_0) \\ (p_2 - c_0 - c_2) \end{bmatrix} = \begin{bmatrix} -G \lambda \left( q_1(a) - q_2(a) \frac{\frac{\partial q_2}{\partial p_1}(a)}{\frac{\partial q_2}{\partial p_2}(a)} \right) - \lambda(1 - G) q_1 \\ -(1 - G) \lambda q_2 \end{bmatrix}$$

or

$$(1 + \lambda) A \begin{bmatrix} p_1 - c_1 - c_0 \\ p_2 - c_0 - c_2 \end{bmatrix} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix}$$

$$\det A = G(1 - G) \frac{\frac{\partial q_2}{\partial p_2}}{\frac{\partial q_2}{\partial p_2}(a)} \Delta(a) + (1 - G)^2 \Delta$$

with

$$\Delta = \frac{\partial q_1}{\partial p_1} \frac{\partial q_2}{\partial p_2} - \frac{\partial q_2}{\partial p_1} \frac{\partial q_1}{\partial p_2}, \quad \Delta(a) = \frac{\partial q_1}{\partial p_1}(a) \frac{\partial q_2}{\partial p_2}(a) - \frac{\partial q_1}{\partial p_2}(a) \frac{\partial q_2}{\partial p_1}(a)$$

$$A^{-1} = \frac{1}{\det A} \begin{bmatrix} (1 - G) \frac{\partial q_2}{\partial p_2} & -(1 - G) \frac{\partial q_1}{\partial p_2} \\ -(1 - G) \frac{\partial q_2}{\partial p_1} & \frac{G \Delta(a)}{\frac{\partial q_2}{\partial p_2}(a)} + (1 - G) \frac{\partial q_1}{\partial p_1} \end{bmatrix}.$$

This yields :

$$p_1 - c_1 - c_0 = \frac{-\lambda \left[ G \left( q_1(a) - q_2(a) \frac{\frac{\partial q_2}{\partial p_1}(a)}{\frac{\partial q_2}{\partial p_2}(a)} \right) + (1 - G) q_1 \right] (1 - G) \frac{\partial q_2}{\partial p_2} + (1 - G)^2 \lambda q_2 \frac{\partial q_1}{\partial p_2}}{(1 + \lambda) \left[ (1 - G)^2 \Delta + G(1 - G) \Delta(a) \left( \frac{\partial q_2}{\partial p_2} / \frac{\partial q_2}{\partial p_2}(a) \right) \right]}$$

$$p_2 - c_2 - c_0 = \frac{\lambda \left[ G \left( q_1(a) - q_2(a) \frac{\frac{\partial q_2}{\partial p_1}(a)}{\frac{\partial q_2}{\partial p_2}(a)} \right) + (1 - G) q_1 \right] (1 - G) \frac{\partial q_2}{\partial p_1} - \lambda(1 - G) q_2 \left[ \frac{G \Delta(a)}{\frac{\partial q_2}{\partial p_2}(a)} + (1 - G) \frac{\partial q_1}{\partial p_1} \right]}{(1 + \lambda) \left[ (1 - G)^2 \Delta + G(1 - G) \Delta(a) \left( \frac{\partial q_2}{\partial p_2} / \frac{\partial q_2}{\partial p_2}(a) \right) \right]}$$

$$\frac{p_1 - c_1 - c_0}{p_1} = \frac{\lambda}{1 + \lambda} \cdot \frac{G\eta_2 \frac{R_1(a)}{R_1} \left(1 + \frac{\eta_{12}(a)}{\eta_2(a)}\right) + (1 - G)(\eta_2 + \eta_{12})}{G\tilde{\Delta}(a) \frac{\eta_2}{\eta_2(a)} \frac{R_1(a)}{R_1} + (1 - G)\tilde{\Delta}} \equiv \frac{\lambda}{1 + \lambda} \frac{1}{\bar{\eta}_1}$$

$$\frac{p_2 - c_2 - c_0}{p_2} = \frac{\lambda}{1 + \lambda} \cdot \frac{\left\{ G \frac{R_1(a)}{R_1} \cdot \left( \eta_{21} + \frac{\eta_{12}(a)\eta_{21}}{\eta_2(a)} + \frac{\tilde{\Delta}(a)}{\eta_2(a)} \right) + (1 - G)(\eta_1 + \eta_{21}) \right\}}{G\tilde{\Delta}(a) \frac{\eta_2}{\eta_2(a)} \frac{R_1(a)}{R_1} + (1 - G)\tilde{\Delta}} \equiv \frac{\lambda}{1 + \lambda} \frac{1}{\bar{\bar{\eta}}_2}$$

with

$$\tilde{\Delta} = \eta_1\eta_2 - \eta_{12}\eta_{21}$$

$$\tilde{\Delta}(a) = \eta_1(a)\eta_2(a) - \eta_{12}(a)\eta_{21}(a).$$

These formulas reduce to the classical superelasticity formulas when  $G = 0$ .

From (A5.3)

$$\begin{aligned} \frac{c_0 - a}{p_2} &= \frac{\lambda}{1 + \lambda} \frac{1}{\eta_2(a)} + \frac{p_1 - c_1 - c_0}{p_1} \frac{\eta_{12}(a)}{\eta_2(a)} \frac{R_1(a)}{R_2(a)} - \frac{p_2 - c_2 - c_0}{p_2} \\ &= \frac{\lambda}{1 + \lambda} \left\{ \frac{1}{\eta_2(a)} + \frac{\eta_{21}(a)}{\bar{\eta}_1\eta_2(a)} - \frac{1}{\bar{\bar{\eta}}_2} \right\}. \end{aligned}$$

## APPENDIX 6 : Implications of the Efficient Component Pricing rule

The regulator's program is then :

$$\max_{\underline{\beta}} \int_{\underline{\beta}}^{\bar{\beta}} \left[ S(q_0(p_0(\beta))) + V(q_1(p_1(\beta), p_1(\beta) - H_1(\beta - e_1(\beta)) + c), q_2(p_1(\beta), p_1(\beta) - H_1(\beta - e_1(\beta)) + c) \right. \\ \left. - p_0(\beta)q_0(p_0(\beta)) - p_1(\beta)q_1(p_1(\beta), p_1(\beta) - H_1(\beta - e_1(\beta)) + c) \right. \\ \left. - (p_1(\beta) - H_1(\beta - e_1(\beta)) + c)q_2(p_1(\beta), p_1(\beta) - H_1(\beta - e_1(\beta)) + c) + U(\beta) \right] dF(\beta)$$

s.t.

$$\dot{U}(\beta) = -\psi'(e_0(\beta) + e_1(\beta)),$$

$$U(\bar{\beta}) \geq 0,$$

and

$$p_0(\beta)q_0(p_0(\beta)) + p_1(\beta)q_1(p_1(\beta), p_1(\beta) - H_1(\beta - e_1(\beta)) + c) \\ + [p_1(\beta) - H_1(\beta - e_1(\beta))]q_2(p_1(\beta), p_1(\beta) - H_1(\beta - e_1(\beta)) + c) \\ - H_0(\beta - e_0(\beta))(q_0(p_0(\beta)) + q_1(p_1(\beta), p_1(\beta) - H_1(\beta - e_1(\beta)) + c) \\ + q_2(p_1(\beta), p_1(\beta) - H_1(\beta - e_1(\beta)) + c)) \\ - H_1(\beta - e_1(\beta))q_1(p_1(\beta), p_1(\beta) - H_1(\beta - e_1(\beta)) + c) - U(\beta) - \psi(e_0(\beta) + e_1(\beta)) = 0.$$

Proceeding as in Section 5 we obtain

$$\psi'(e_0 + e_1) = H'_0(q_0 + q_1 + q_2) - \frac{\int_{\underline{\beta}}^{\bar{\beta}} \tilde{\lambda}(\tilde{\beta})f(\tilde{\beta})d\tilde{\beta}}{(1 + \tilde{\lambda}(\beta))f(\beta)}\psi''(e_0 + e_1) \\ \psi'(e_0 + e_1) = H'_1q_1 - \frac{\int_{\underline{\beta}}^{\bar{\beta}} \tilde{\lambda}(\tilde{\beta})f(\tilde{\beta})d\tilde{\beta}}{(1 + \tilde{\lambda}(\beta))f(\beta)}\psi''(e_0 + e_1) + \left[ \frac{\tilde{\lambda}(\beta)}{1 + \tilde{\lambda}(\beta)}q_2 \right. \\ \left. + (p_1 - c_0 - c_1)\frac{\partial q_1}{\partial p_2} + (p_2 + c - c_0)\frac{\partial q_2}{\partial p_2} \right] H'_1 \\ \frac{p_1 - C_{0Q} - C_{1q_1}}{p_1} = \frac{\tilde{\lambda}(\beta)}{1 + \tilde{\lambda}(\beta)} \left( \frac{-(q_1 + q_2)}{\frac{\partial(q_1 + q_2)}{\partial p_1} + \frac{\partial(q_1 + q_2)}{\partial p_2}} \right) \\ = \frac{\tilde{\lambda}(\beta)}{1 + \tilde{\lambda}(\beta)} \frac{1}{\eta^{**}},$$

with

$$\begin{aligned}\eta^{**} &= \left(\frac{q_1}{q_1 + q_2}\right)\eta_1 + \left(\frac{p_1}{p_2}\right)\left(\frac{q_2}{q_1 + q_2}\right)\eta_2 \\ &\quad - \left(\frac{q_2}{q_1 + q_2}\right)\eta_{21} - \frac{p_1}{p_2}\left(\frac{q_1}{q_1 + q_2}\right)\eta_{12}.\end{aligned}$$

Hence

$$\begin{aligned}a(\beta) &= p_1(\beta) - H_1(\beta - e_1(\beta)) \\ &= C_{0Q} + \frac{\tilde{\lambda}(\beta)}{1 + \tilde{\lambda}(\beta)} \frac{p_1}{\eta^{**}} = C_{0Q} + \frac{\tilde{\lambda}(\beta)}{1 + \tilde{\lambda}(\beta)} \frac{p_2}{\hat{\eta}_2^*}\end{aligned}$$

with  $\hat{\eta}_2^* = \frac{p_2}{p_1}\eta^{**}$ .



## APPENDIX 7 : The Efficient Component Pricing rule under price caps

The Lagrangian of the monopoly's optimization problem is

$$\begin{aligned} & p_0 q_0(p_0) - H_0(\beta - e_0)(q_0(p_0) + q_1(p_1, p_1 - H_1(\beta - e_1(\beta)) + c) + q_2(p_1, p_1 - H_1(\beta - e_1(\beta)) + c)) \\ & + (p_1 - H_1(\beta - e_1)) (q_1(p_1, p_1 - H_1(\beta - e_1(\beta)) + c) + q_2(p_1, p_1 - H_1(\beta - e_1(\beta)) + c)) \\ & - \psi(e_0 + e_1) + \nu(\bar{p} - \alpha_0 p_0 - \alpha_1 p_1). \end{aligned}$$

The first-order conditions with respect to  $p_0$  and  $p_1$  are

$$\begin{aligned} & (p_0 - c_0) \frac{dq_0}{dp_0} + q_0 = \alpha_0 \nu \\ & (p_1 - c_0 - c_1) \left( \frac{\partial q_1}{\partial p_1} + \frac{\partial q_1}{\partial p_2} + \frac{\partial q_2}{\partial p_1} + \frac{\partial q_2}{\partial p_2} \right) + q_1 + q_2 = \alpha_1 \nu. \end{aligned}$$

If  $\alpha_0 \simeq q_0$  and  $\alpha_1 \simeq q_1 + q_2$ ,

$$\frac{p_0 - c_0}{p_0} \approx \frac{1 - \nu}{\eta_0}$$

$$\frac{p_1 - c_0 - c_1}{p_1} \approx \frac{1 - \nu}{\hat{\eta}_2^*} \frac{p_2}{p_1}$$

where  $\hat{\eta}_2^*$  is given by (47).

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